

Statistical density modification in macromolecular crystallography

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Statistical density modification (RESOLVE)

- Principle: phase probability information from probability of the map and from experiment:

- $P(\phi) = P_{\text{map probability}}(\phi) P_{\text{experiment}}(\phi)$

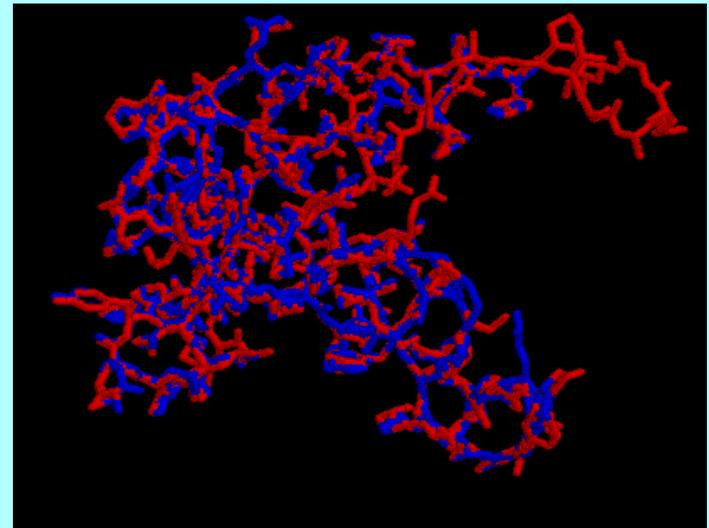
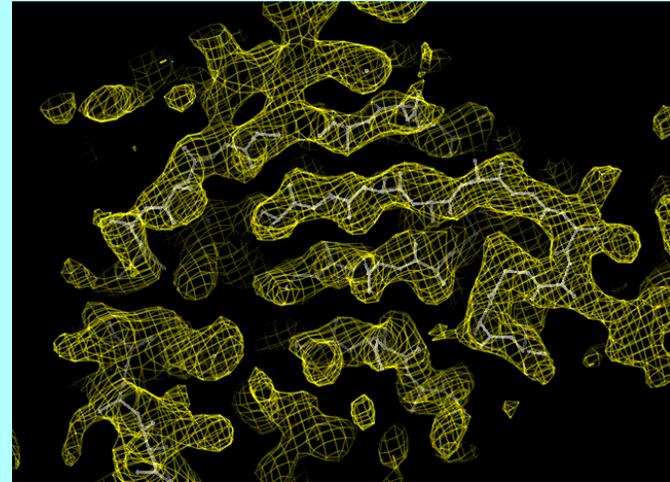
- “Phases that lead to a believable map are more probable than those that do not”

- **A believable map is a map that has...**

- a relatively flat solvent region
- NCS (if appropriate)
- A distribution of densities like those of model proteins

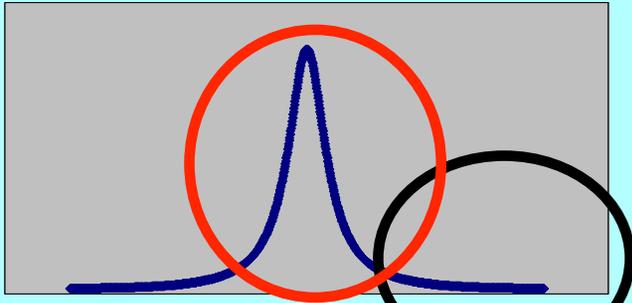
- **Method:**

- calculate how map probability varies with electron density ρ
- deduce how map probability varies with phase ϕ
- combine with experimental phase information

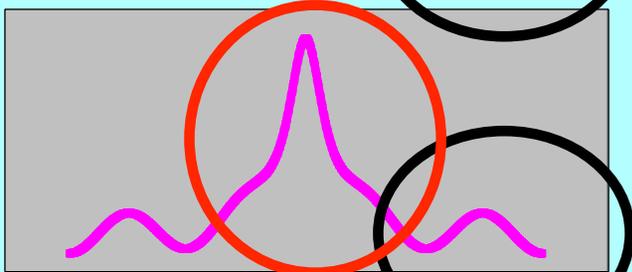


Map probability phasing: Getting a new probability distribution for each phase given estimates of all others

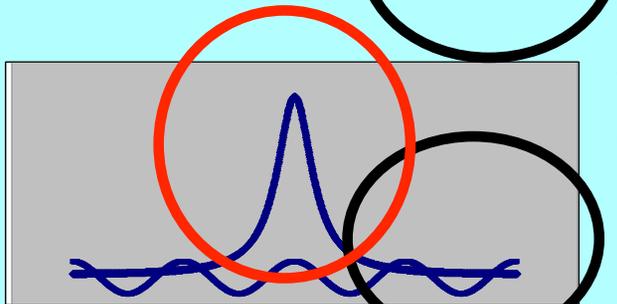
1. Identify expected features of map (flat far from center)
2. Calculate map with current estimates of all structure factors except one (k)
3. Test all possible phases ϕ for structure factor k (for each phase, calculate new map including k)
4. Probability of phase ϕ estimated from agreement of map with expectations
5. **Phase probability of reflection k from map is independent of starting phase probability because reflection k is omitted from the map**



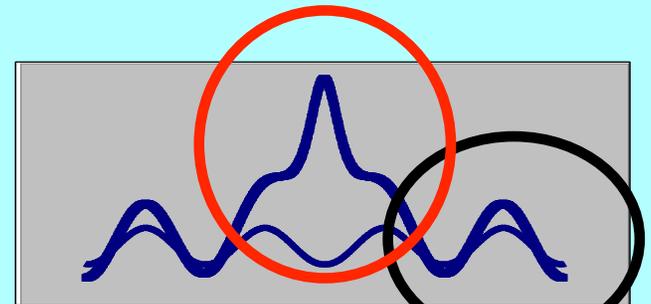
A function that is (relatively) flat far from the origin



Function calculated from estimates of all structure factors but one (k)



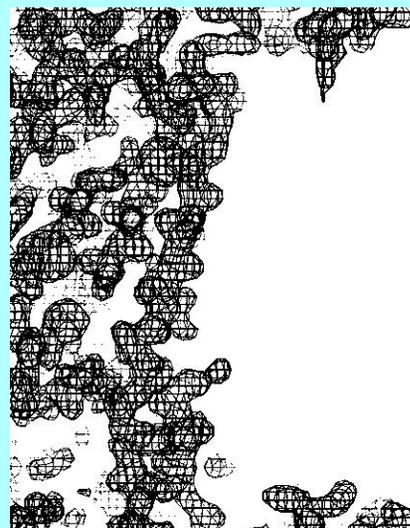
Test each possible phase of structure factor k . $P(\phi)$ is high for phase that leads to flat region



A map-probability function – allowing different weighting of information from different parts of the map

Log-probability of the map is sum over all points in map of local log-probability

$$LL^{MAP}(\{\mathbf{F}_h\}) \approx \frac{N_{REF}}{V} \int_V LL(\rho(\mathbf{x}, \{\mathbf{F}_h\})) d^3\mathbf{x}$$



A map with a flat (blank) solvent region is a likely map

Local log-probability is believability of the value of electron density ($\rho(\mathbf{x})$) found at this point

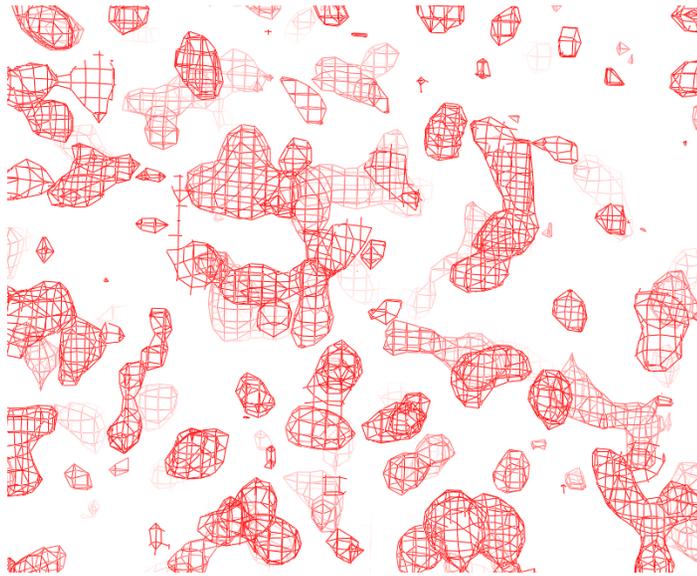
$$LL(\rho(\mathbf{x}, \{\mathbf{F}_h\})) = \ln[p(\rho(\mathbf{x})|PROT)p_{PROT}(\mathbf{x}) + p(\rho(\mathbf{x})|SOLV)p_{SOLV}(\mathbf{x})]$$

If the point is in the PROTEIN region, most values of electron density ($\rho(\mathbf{x})$) are believable

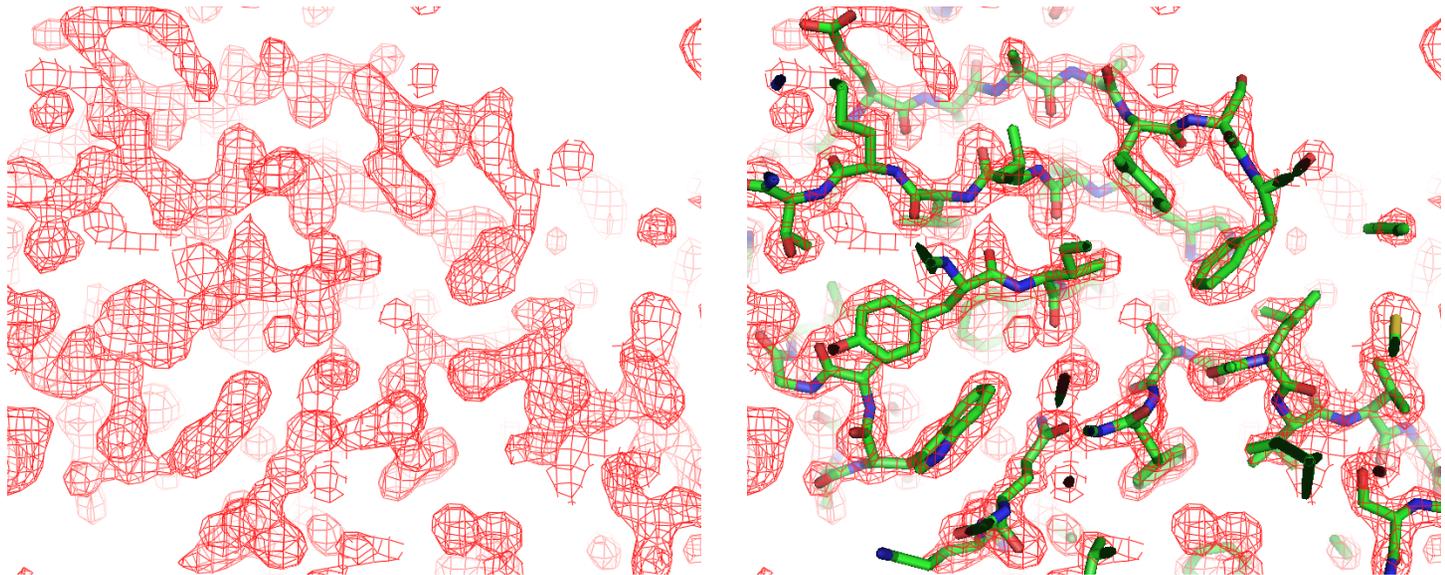
If the point is in the SOLVENT region, only values of electron density near zero are believable

Statistical density modification (nsf-N SAD map , 2Å, no NCS, 50% solvent)

Phaser SAD map
(CC=0.43)



Phaser +RESOLVE
(CC=0.79)



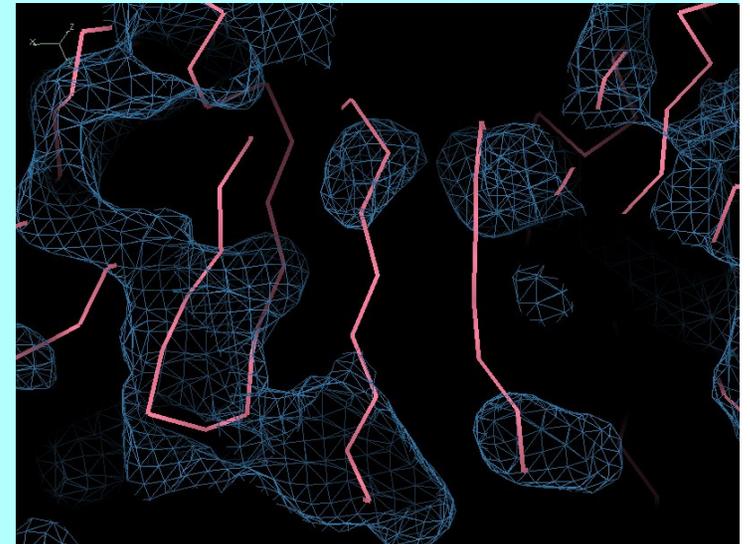
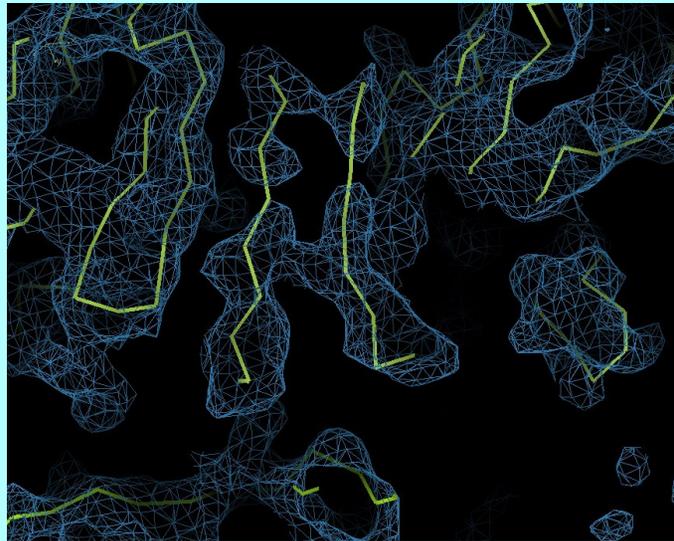
Statistical density modification with cross-crystal averaging

Cell receptor at 3.5/3.7 Å. Data courtesy of J. Zhu

Crystal 1 (4 copies)

Crystal 2 (2 copies)

RESOLVE
density
modification



PHENIX
Multi-crystal
averaging

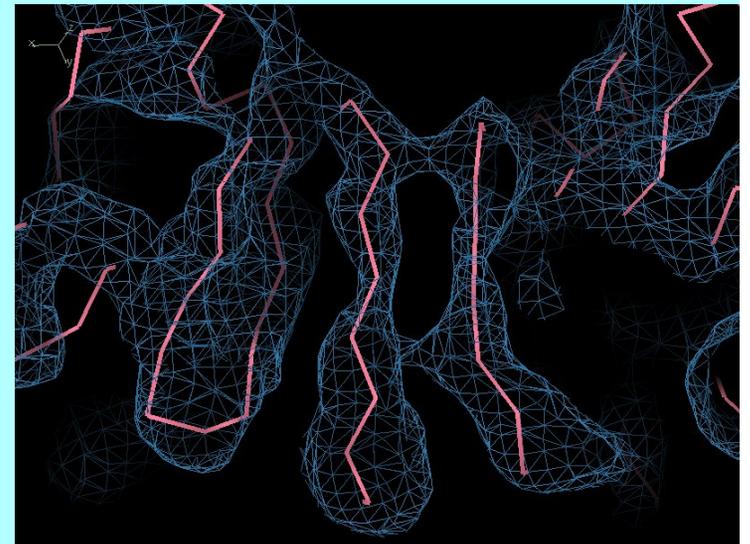
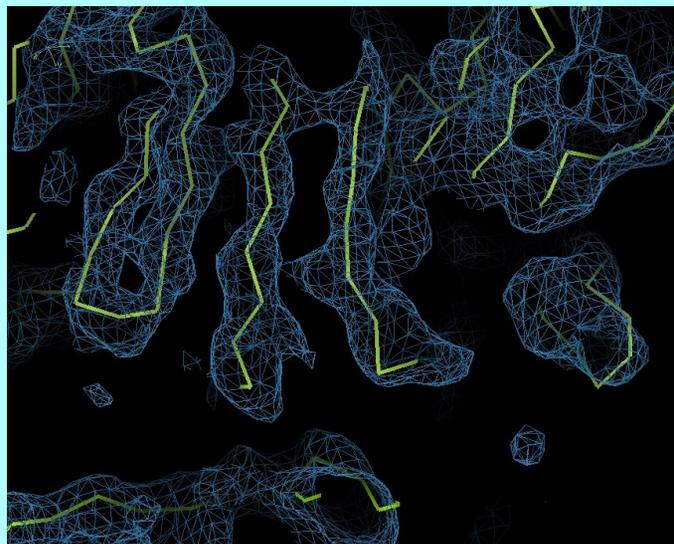


Image enhancement using local feature recognition

Electron density maps of proteins have many features in common

- Connected density
- Preferred distances for spacing between regions of high density
- Preferred shapes of density

Starting image in red

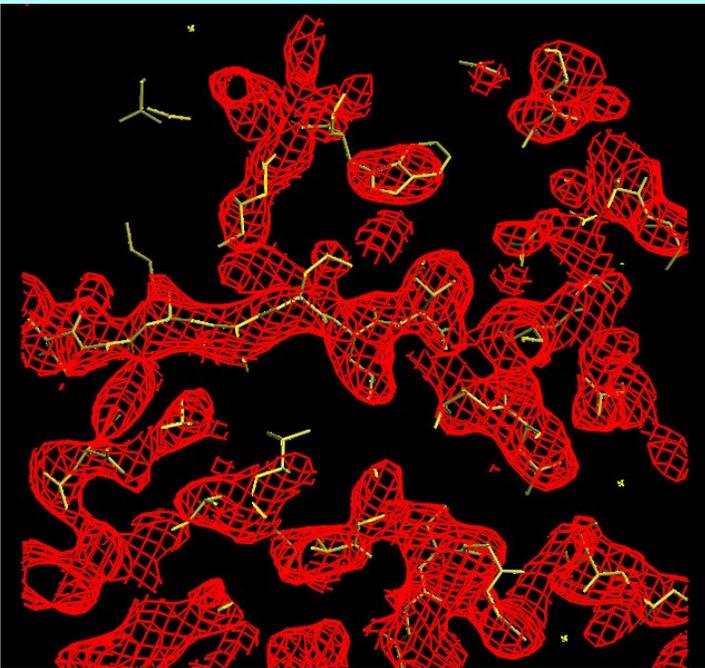


Image improved using expectations about local features

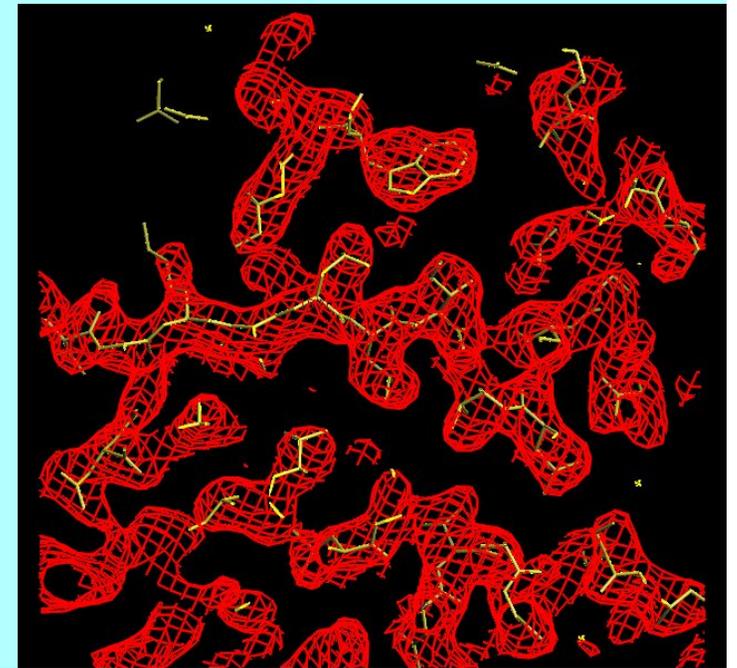


Image enhancement using local feature recognition

Approach:

- Use the pattern of density **near** a point x to estimate the value of density **at** x
- Combine new estimate of density with previous one to improve the overall image

Starting image in red

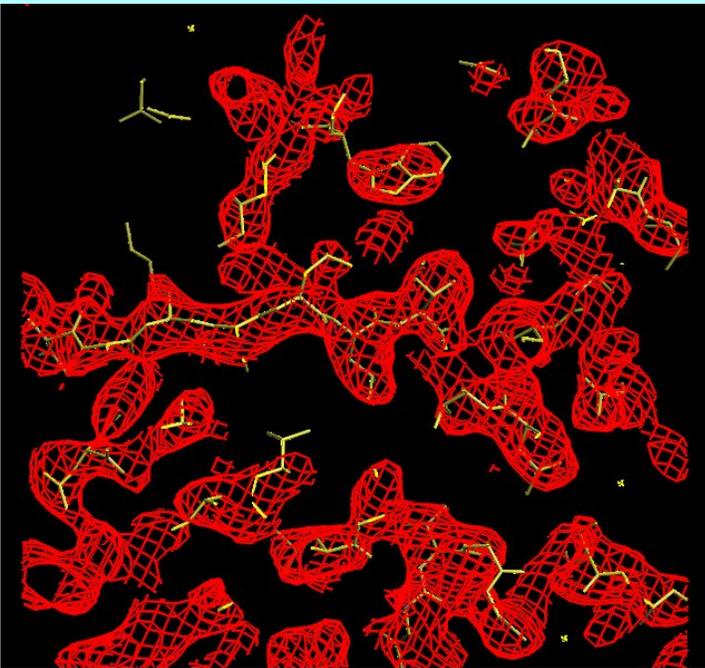


Image improved using expectations about local features

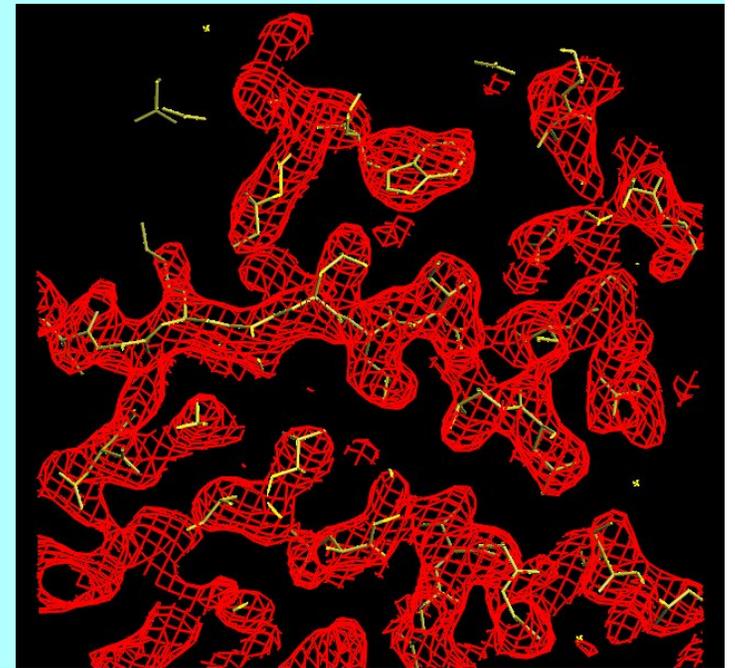


Image enhancement using local feature recognition

Approach:

- Create N templates of local density using model data
 - Examine density near each point x in image (within 2 Å)
 - Exclude region very close to x (about 1 Å)
 - Cluster and average local patterns of density (after rotation to maximize CC)
- Identify relationship between finding pattern k of density near x , and density at x
 - Find all locations in the image where template k best matches the local density near x
 - Calculate average value of density at x for these cases = $\rho_{\text{mean}}(k)$
- Identify pattern near each point in actual map and use it to estimate density at that point
 - For each point x in the image, identify which template k best matches the local density near x
 - Use $\rho_{\text{mean}}(k)$ as estimate of density at x

Constructing a feature library

Identify averaged local features of model images

Go through map point-by-point. For each point x (marked by * at center of pictures)

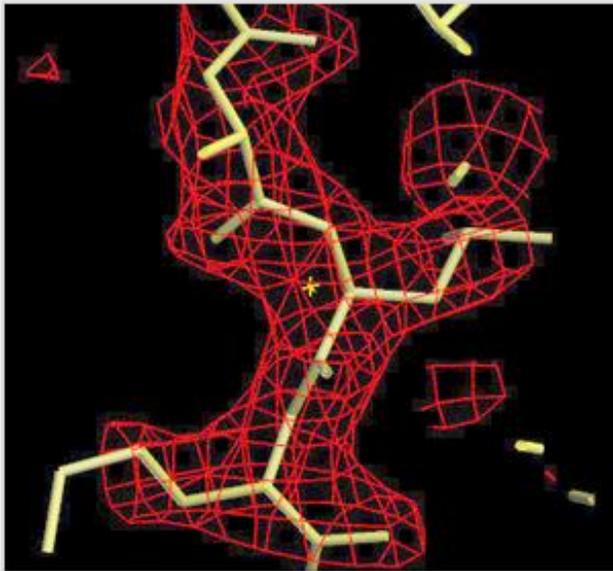
Examine density in map near x : $\rho_{\text{current}}(x + \Delta x)$

Remove all information about density at x from $\rho(x + \Delta x)$

-> $g(x + \Delta x)$, local pattern at x , without information from x

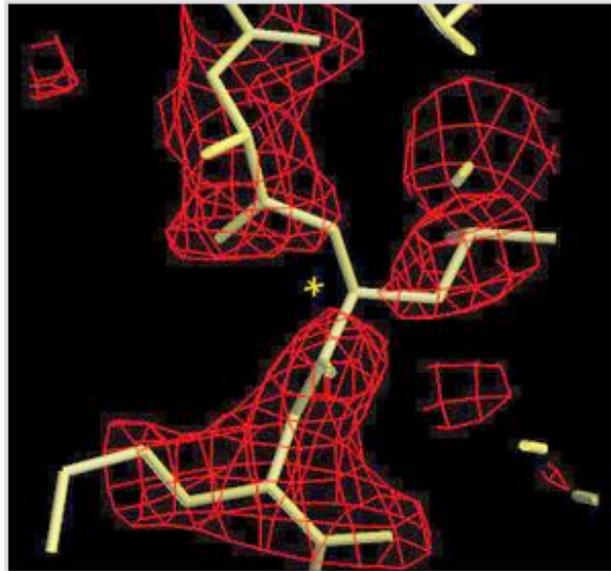
Group local density patterns by correlation coefficient into clusters

Sort clusters according to average density at x for that cluster



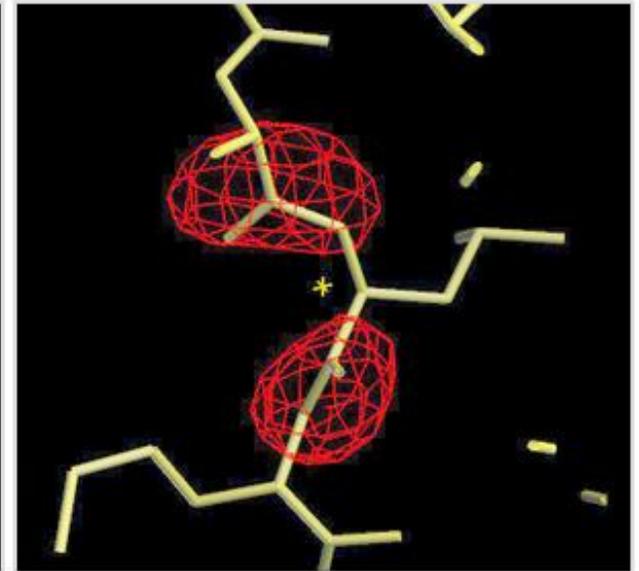
$$\rho_{\text{current}}(x + \Delta x)$$

Local density near x in model image



$$g(x + \Delta x)$$

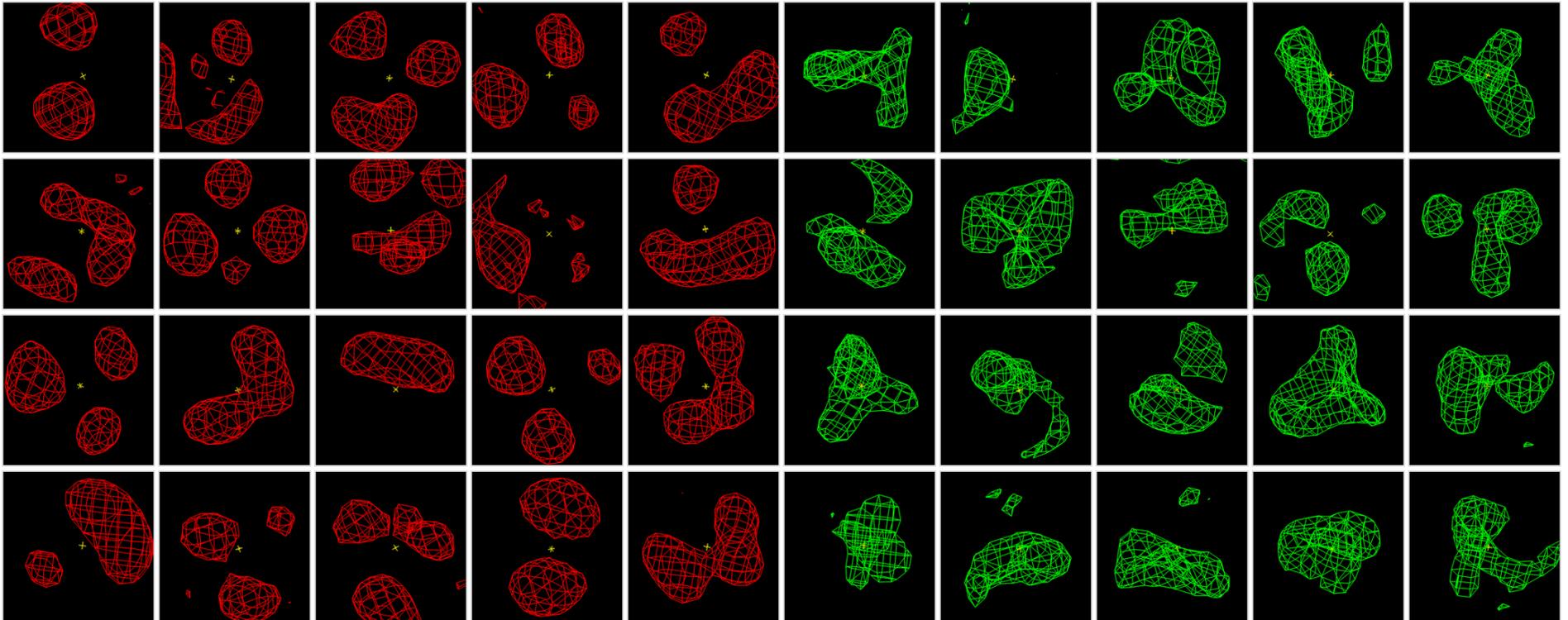
local density... setting points near x to near zero



Average of local density for one cluster associated with positive density at x

Constructing a feature library

20 Templates of average local density



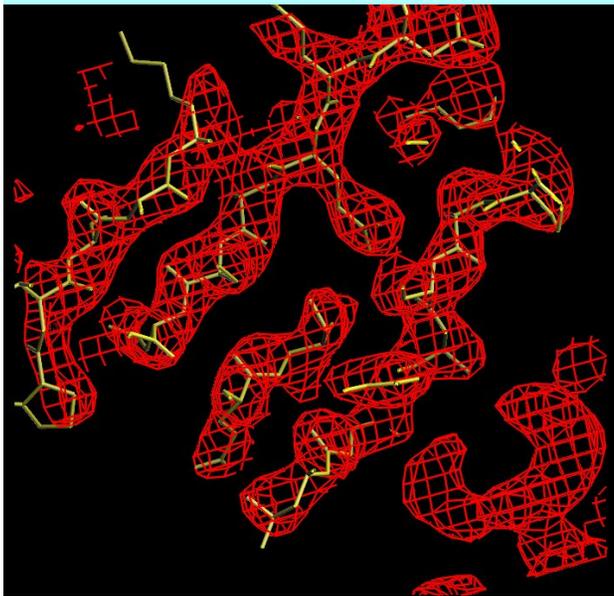
Contours at $+1.5 \sigma$

Contours at -1.5σ

Image enhancement using local feature recognition

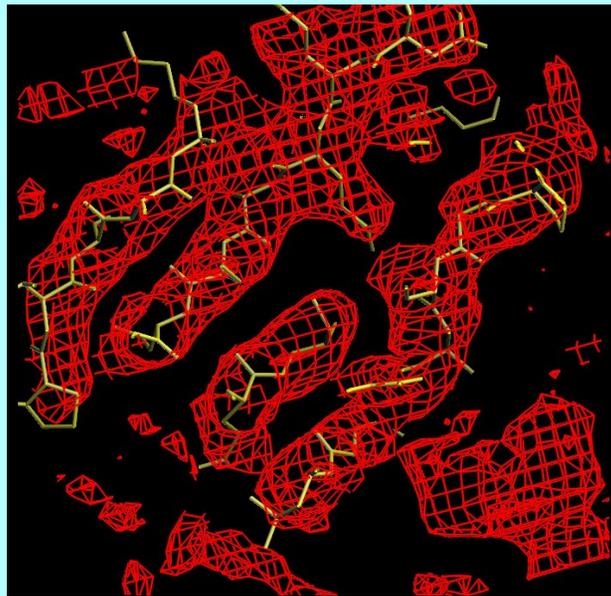
Image recovery from a good map...

- gives an image that has (mostly) correct features
- errors are (almost) uncorrelated with original errors



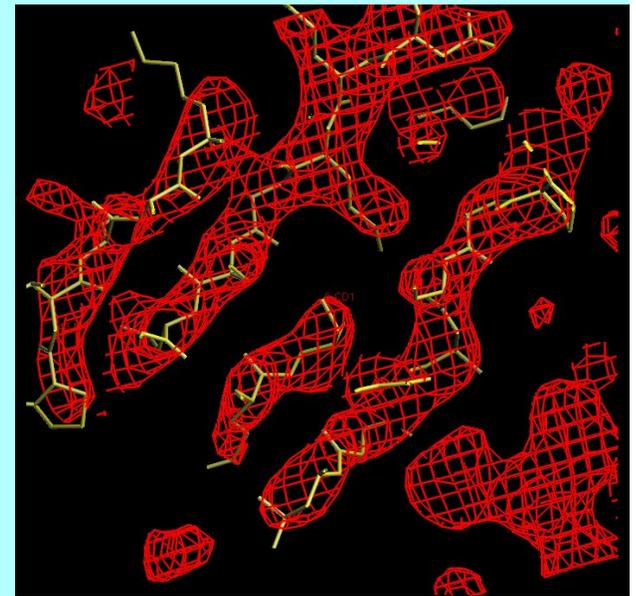
RESOLVE map gene 5
protein at 2.6 Å

CC to perfect map = 0.8



Recovered image
derived from RESOLVE map

CC to perfect map = 0.36



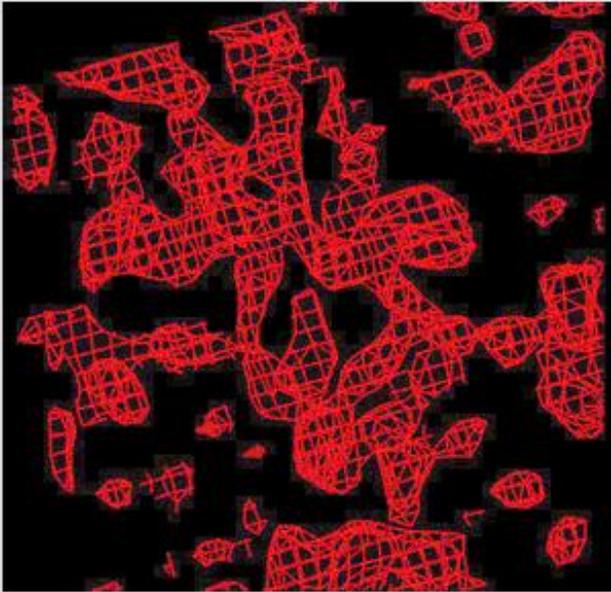
Map phased using only
information from
recovered image

CC to perfect map = 0.64

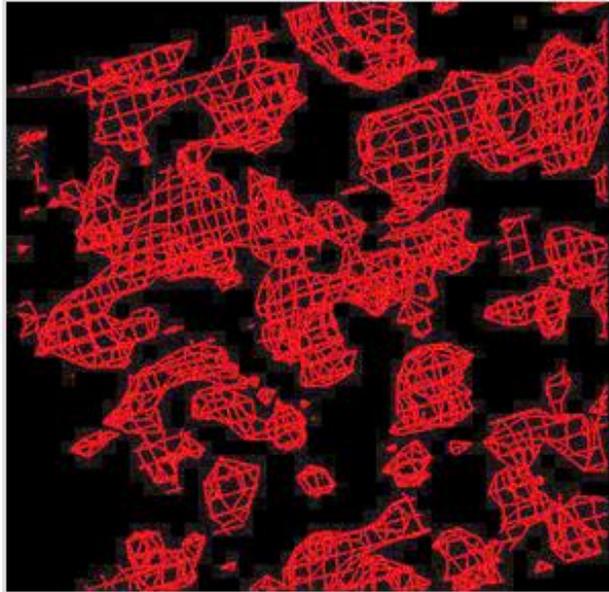
CC of errors with errors in
RESOLVE map = 0.11

Image enhancement using local feature recognition

Image recovery from a random map...gives an uncorrelated image

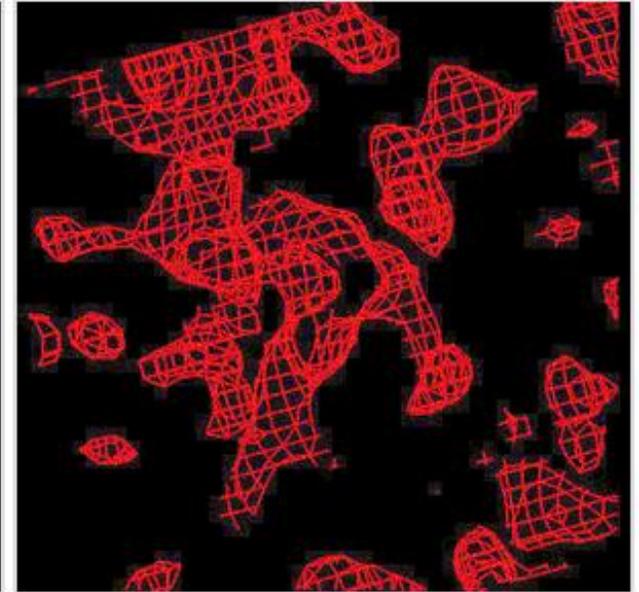


Random map at 2.6 Å



Recovered image
derived from random map

CC to original
random map=-0.01



Map phased using only
recovered image

CC to original
random map=-0.04

Iterative procedure for image enhancement using local feature recognition

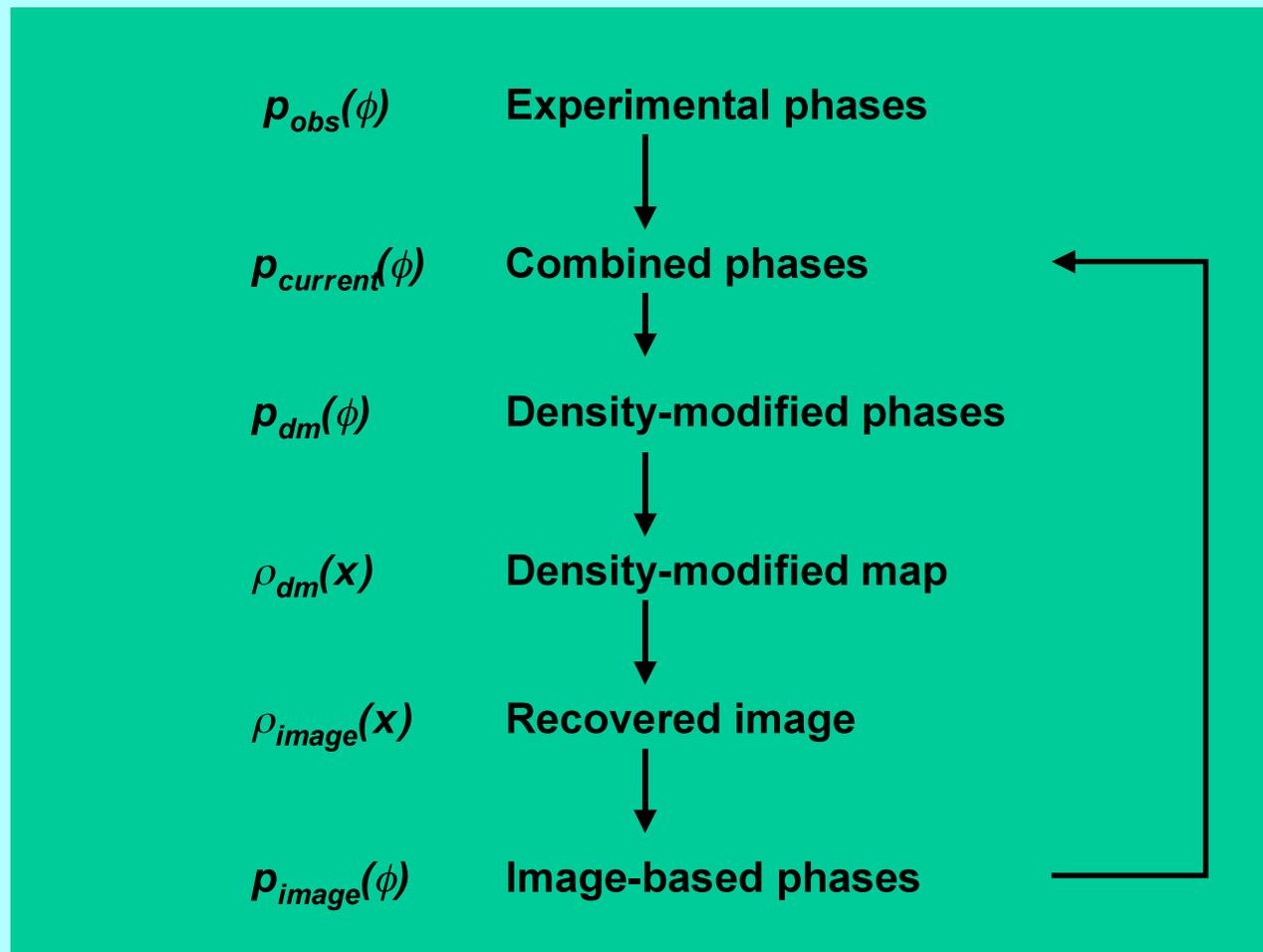
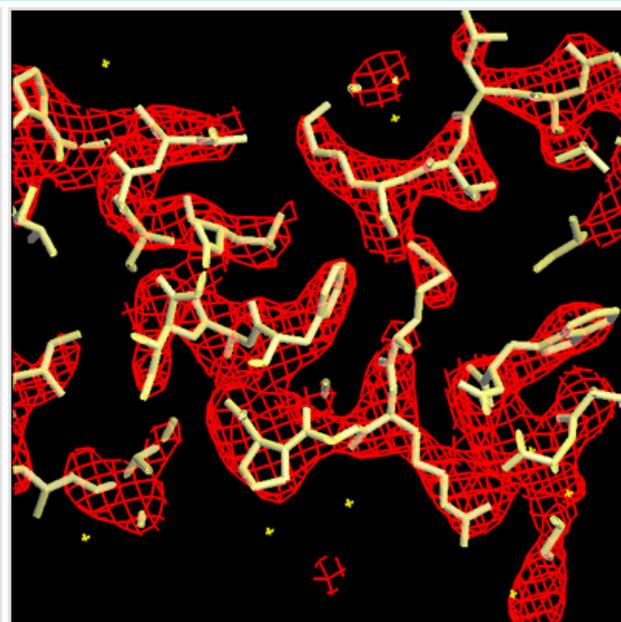
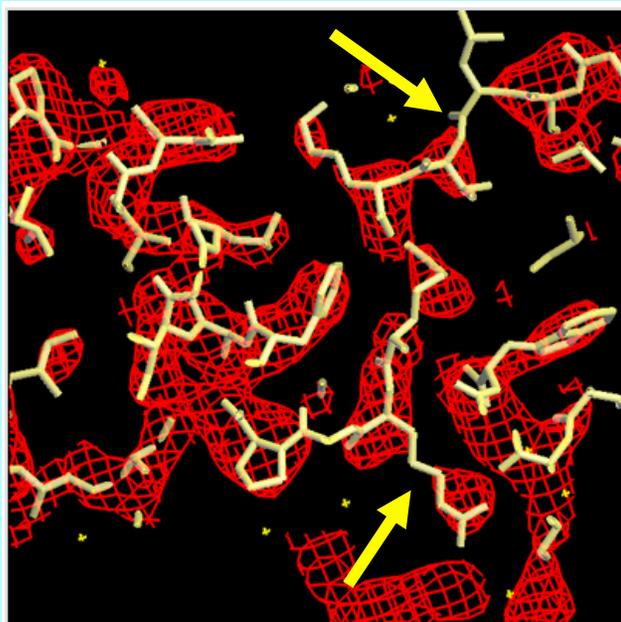


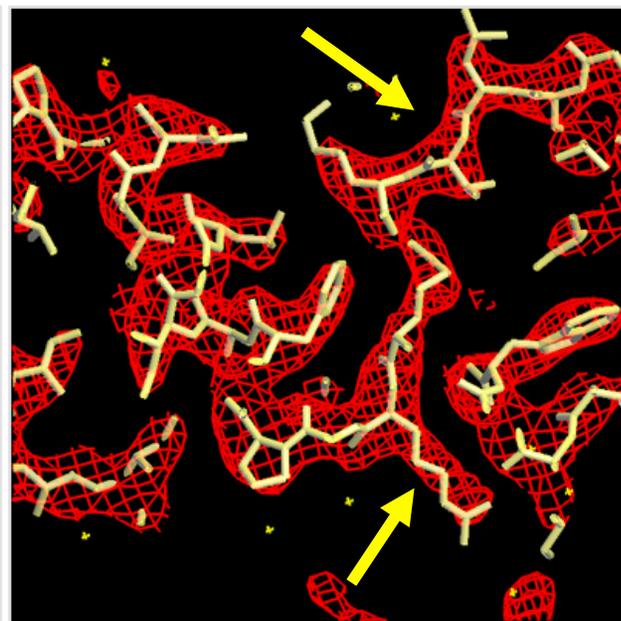
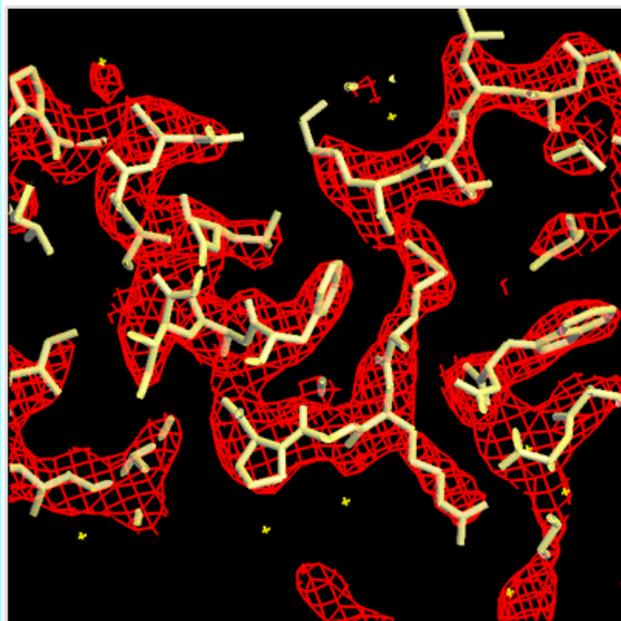
Image enhancement using local feature recognition (nusA protein structure)

Starting
map
CC=0.65



Cycle 1
CC=0.75

Cycle 3
CC=0.84



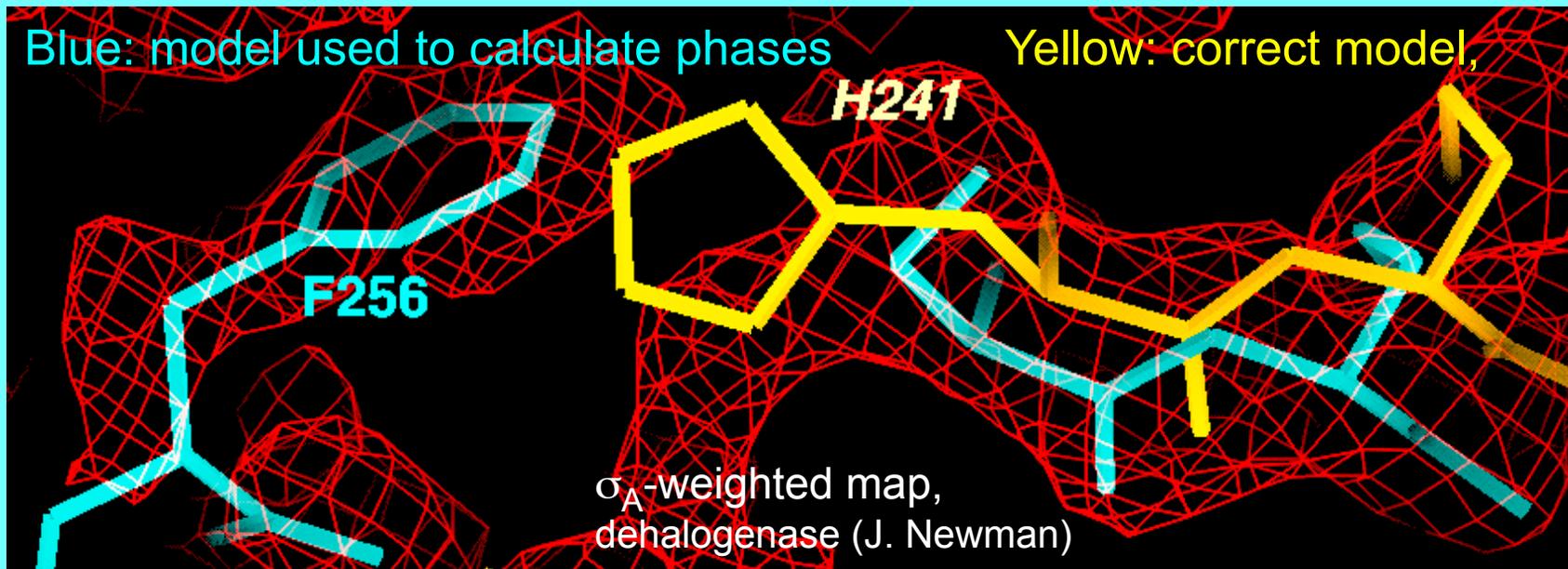
Cycle 5
CC=0.85

Removing model bias with prime-and-switch phasing

The problem:

Atomic model used to calculate phases \rightarrow map looks like the model

Best current solution: σ_A -weighted phases



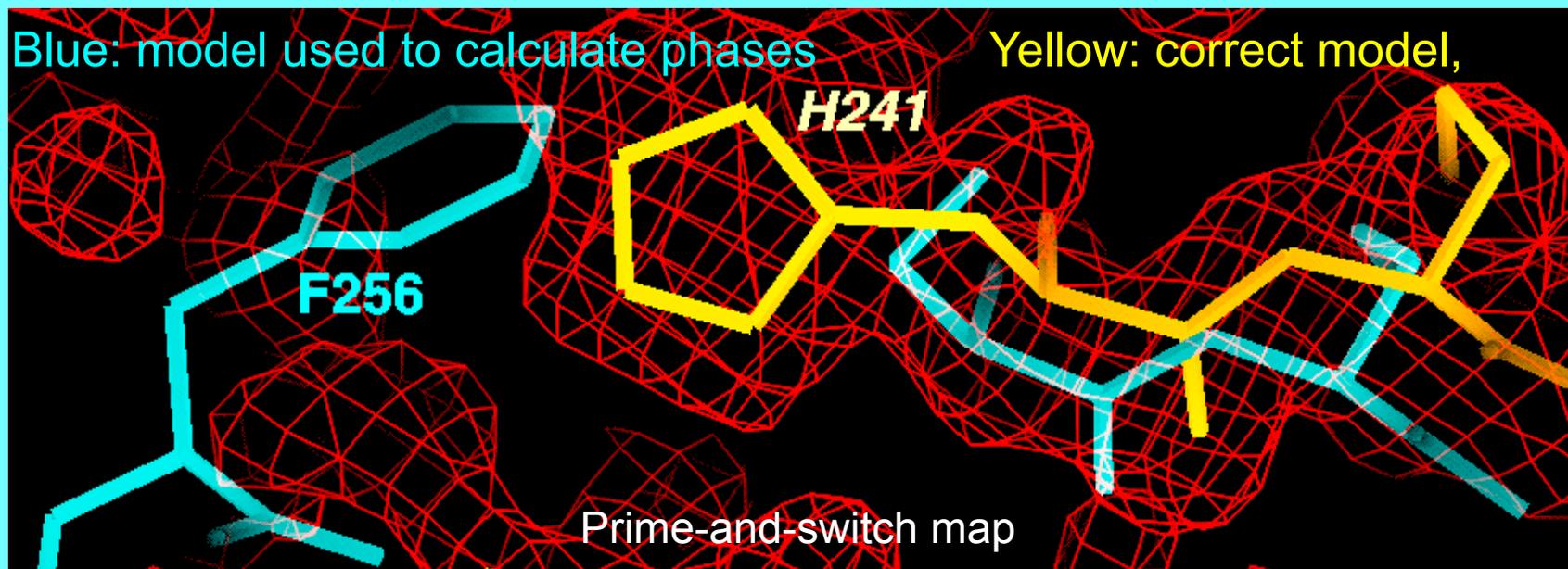
Prime-and-switch phasing

A solution:

Start with σ_A -weighted map

Identify solvent region (or other features of map)

Adjust the phases to maximize the probability of the map – **without biasing towards the model phases**

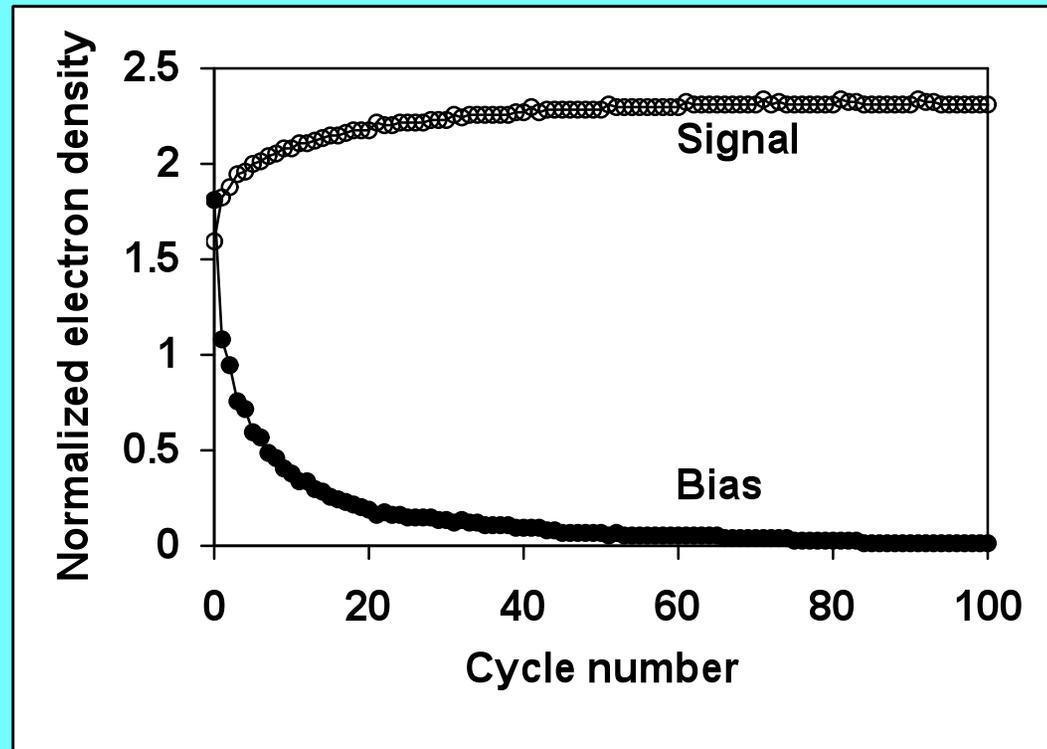


Prime-and-switch phasing

Why it should work...

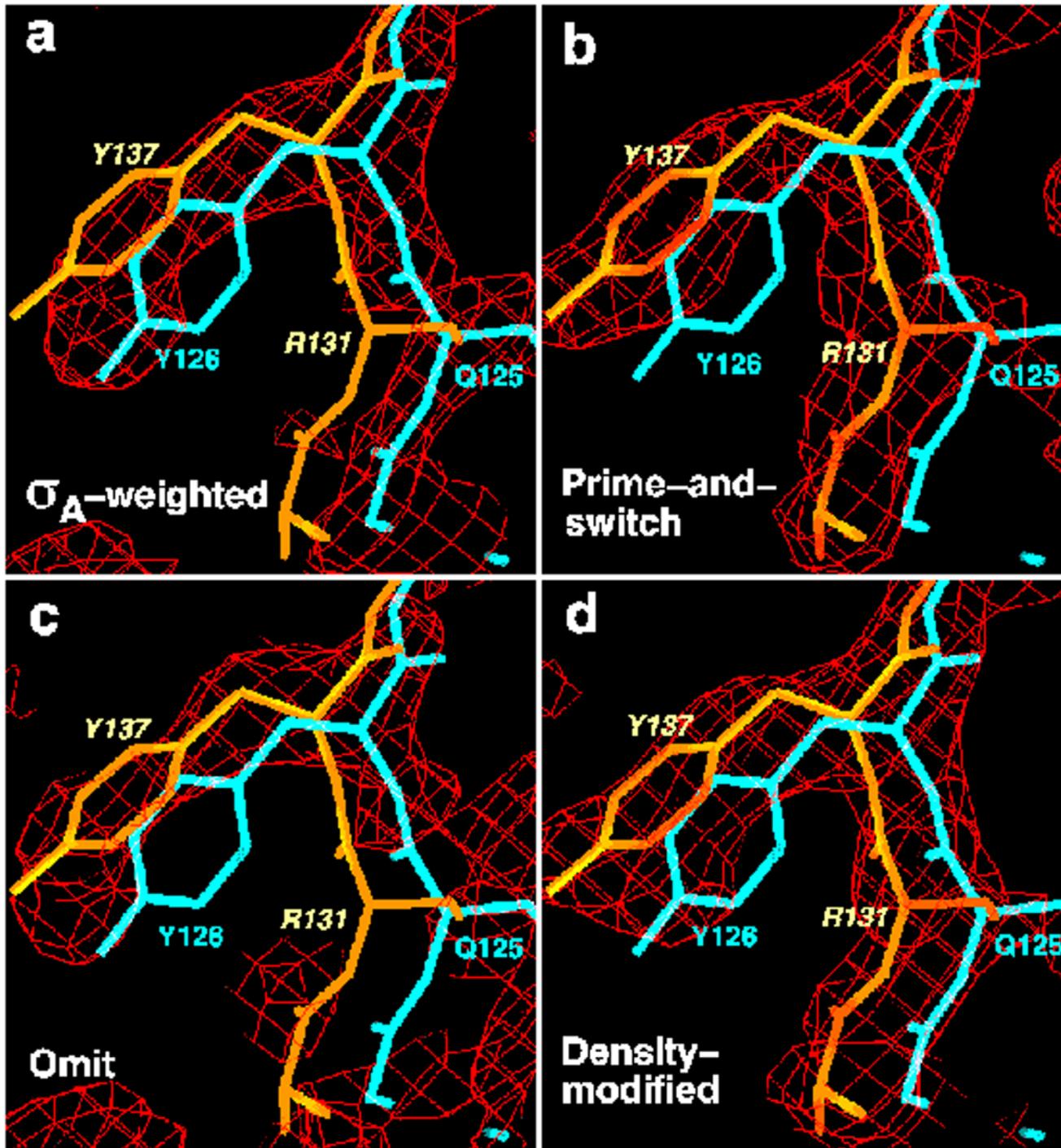
Priming: Starting phases are close to correct ones...but have bias towards misplaced atoms

Switching: Map-probability phase information comes from a different source...which reinforces just the correct phase information



Signal: peak height at correct atomic positions

Bias: peak height at incorrect atoms in starting model



*Prime-and-switch
example*

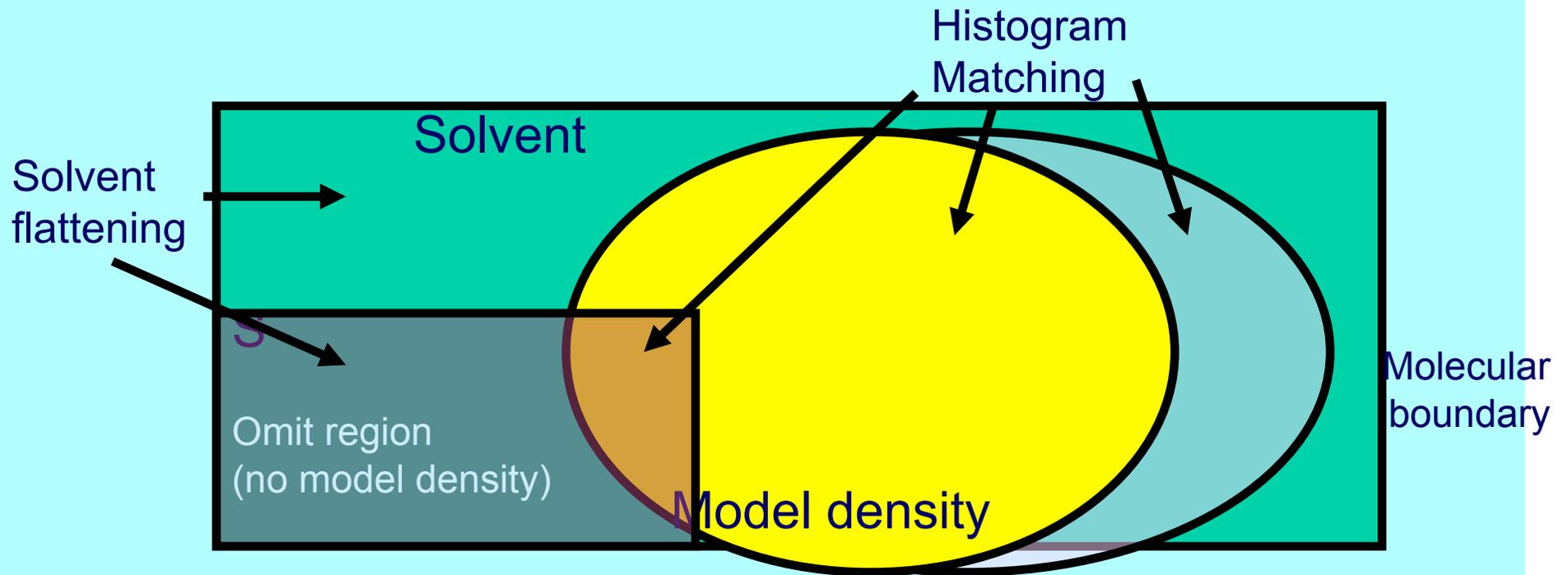
(IF5A, T. Peat)

Orange:
correct model

Blue: model used
to calculate
phases

Composite omit map with statistical density modification

Statistical density modification allows a separate probability distribution for electron density at each point in the map: can specify that “missing” density is within molecular boundary

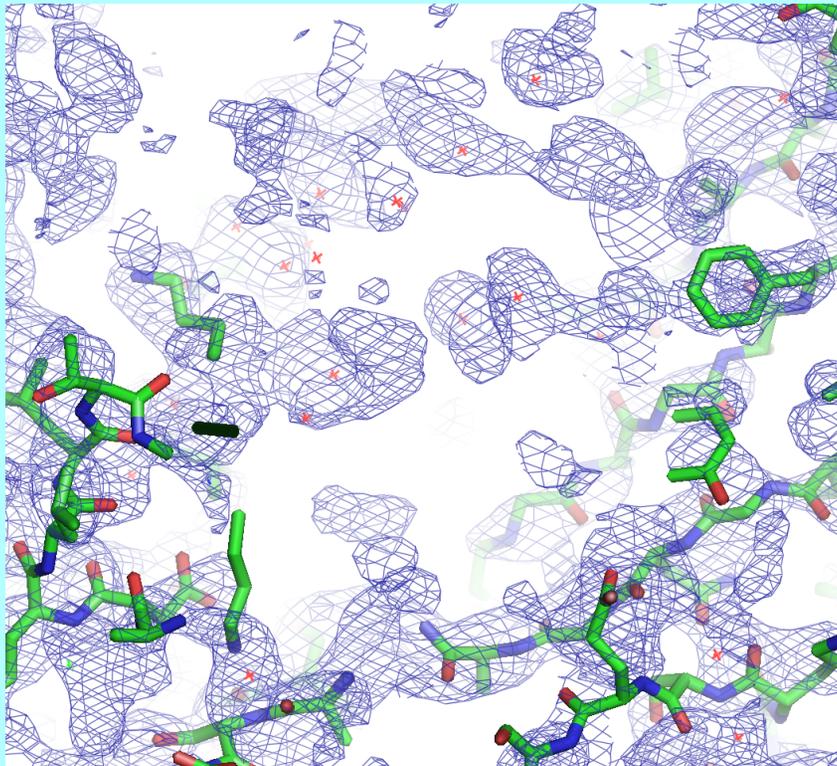


Can be used with or without experimental phases...with or without omit

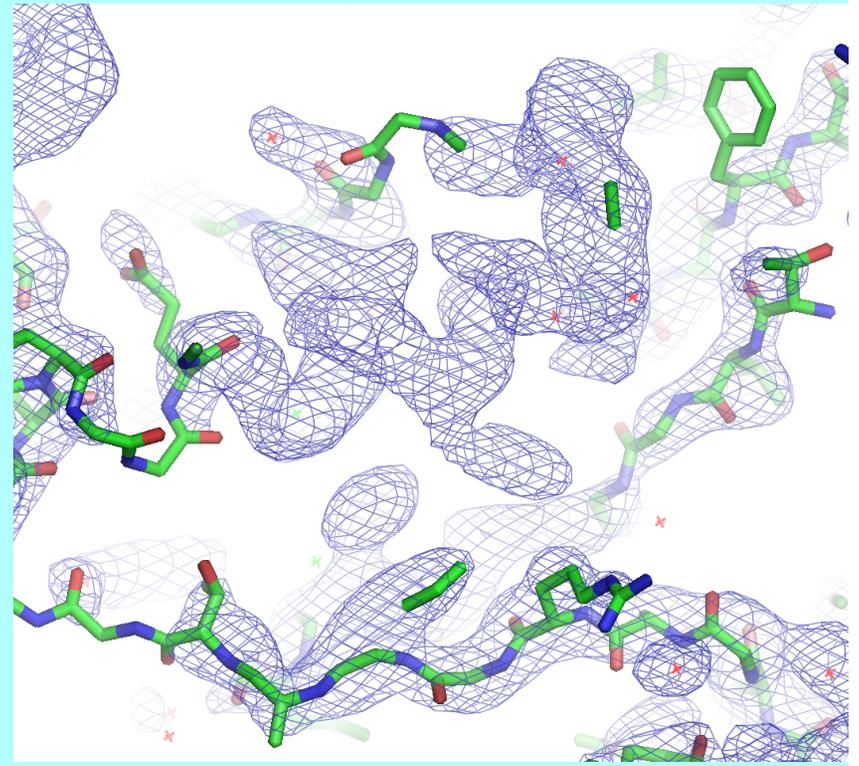
Iterative-Build OMIT procedure

“Is the density in my map biased by the model?”

2mFo-DFc omit map



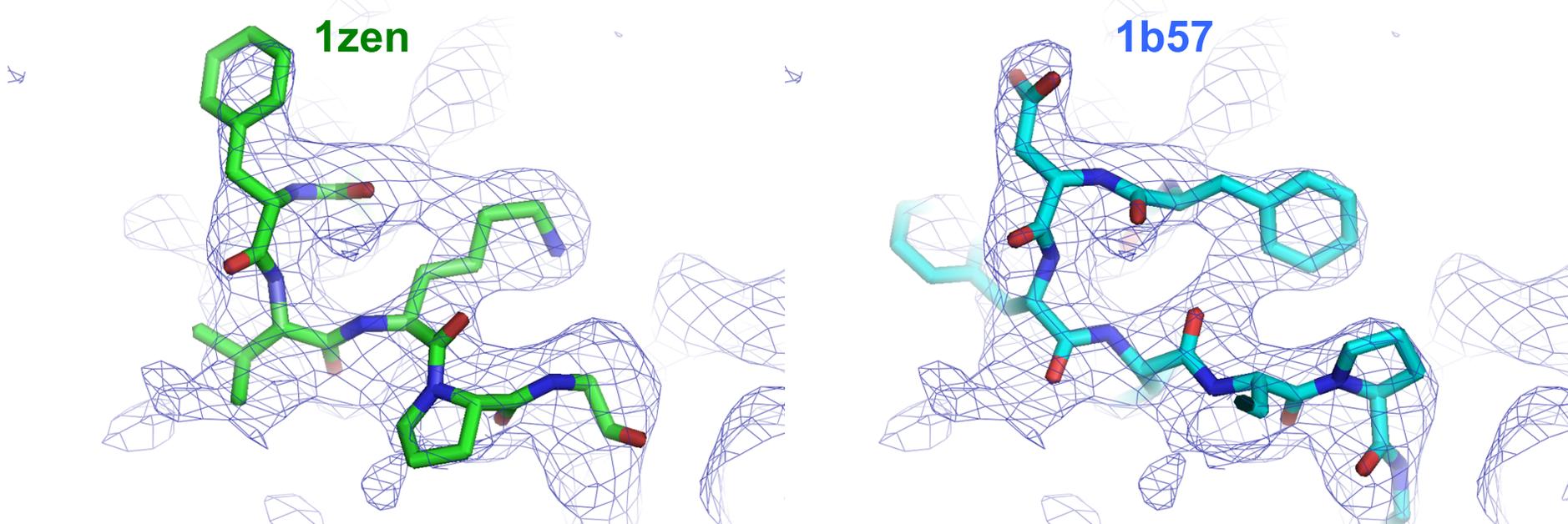
After building outside
OMIT region 10 cycles



1HP7 molecular replacement with 1AS4
R/Rfree after initial refinement: 0.41/0.48

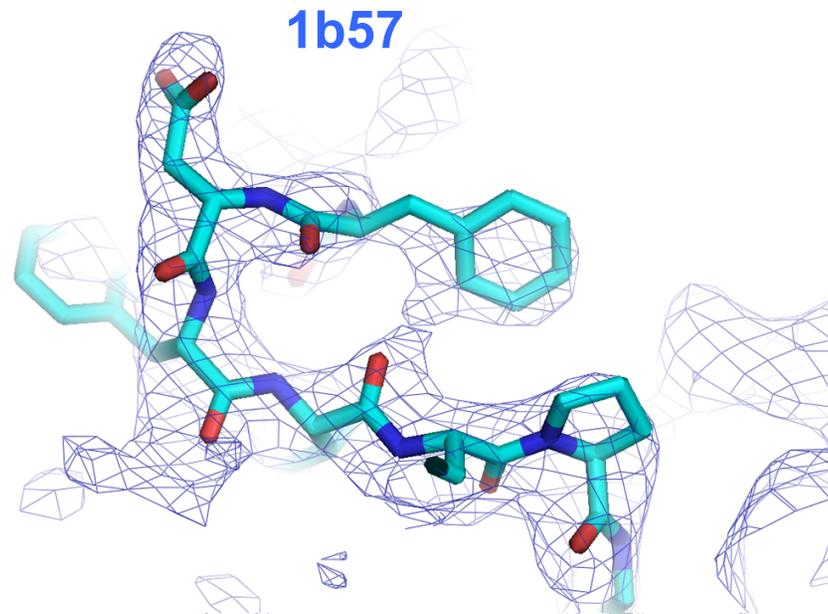
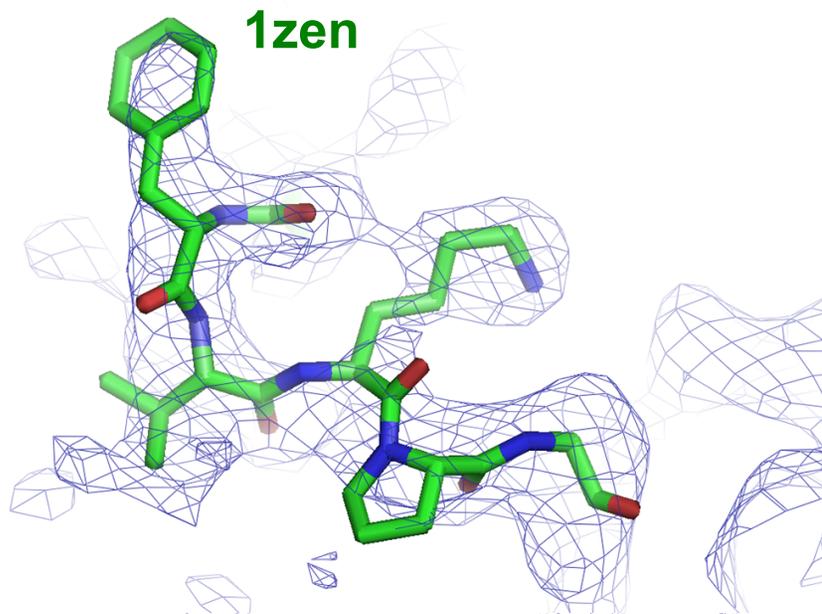
Iterative-Build OMIT procedure “Removing model bias”

2mFo-DFc map
Phased with 1zen model



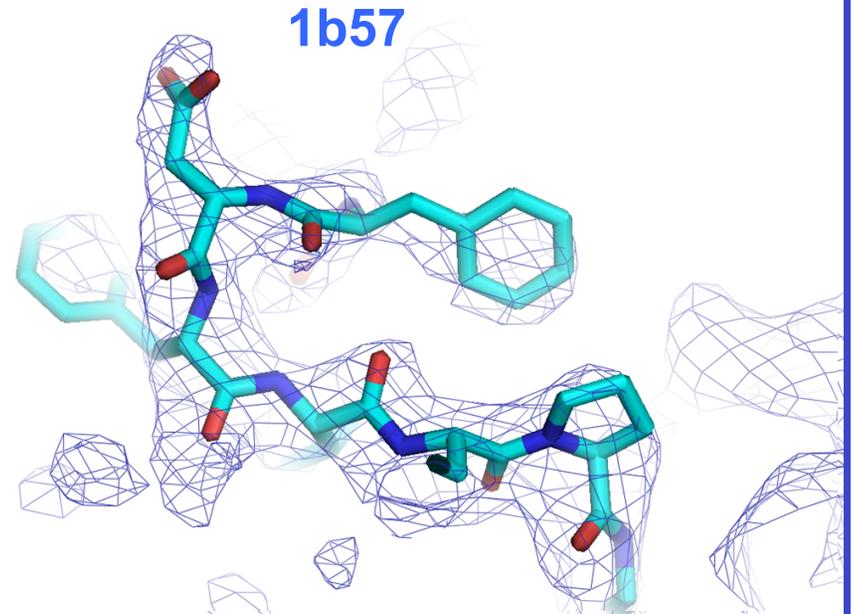
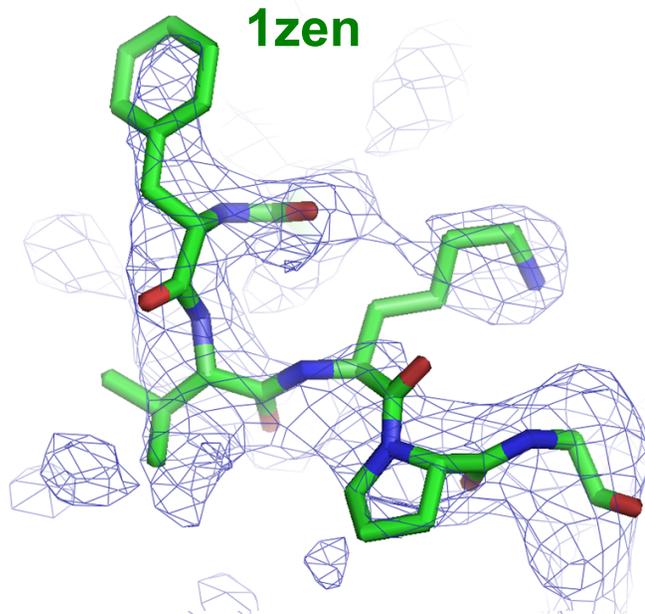
Iterative-Build OMIT procedure “Removing model bias”

2mFo-DFc omit map
Phased with 1zen model



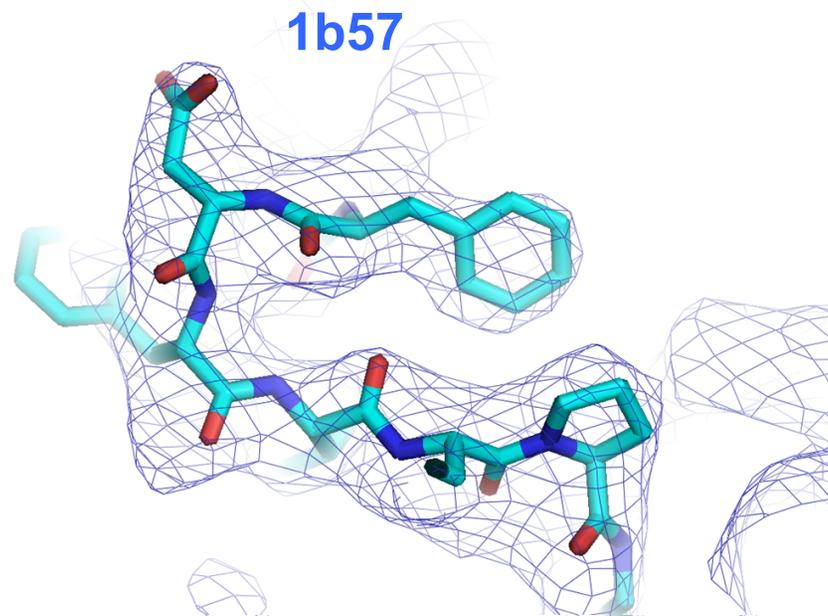
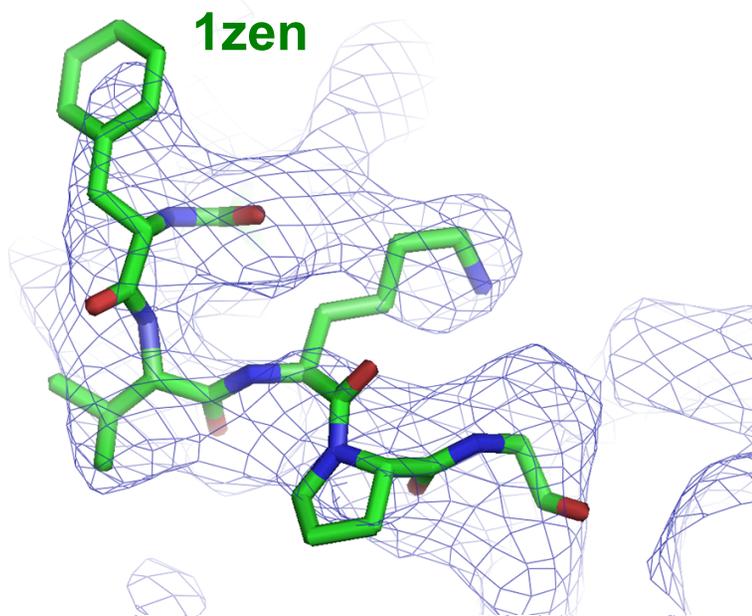
Iterative-Build OMIT procedure “Removing model bias”

2mFo-DFc SA-omit map
Phased starting with 1zen model



Iterative-Build OMIT procedure “Removing model bias”

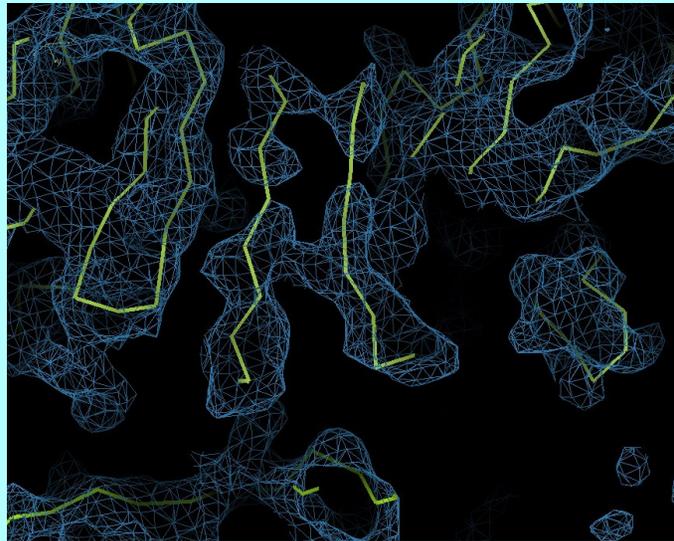
2mFo-DFc iterative-build omit map
Phased starting with 1zen model



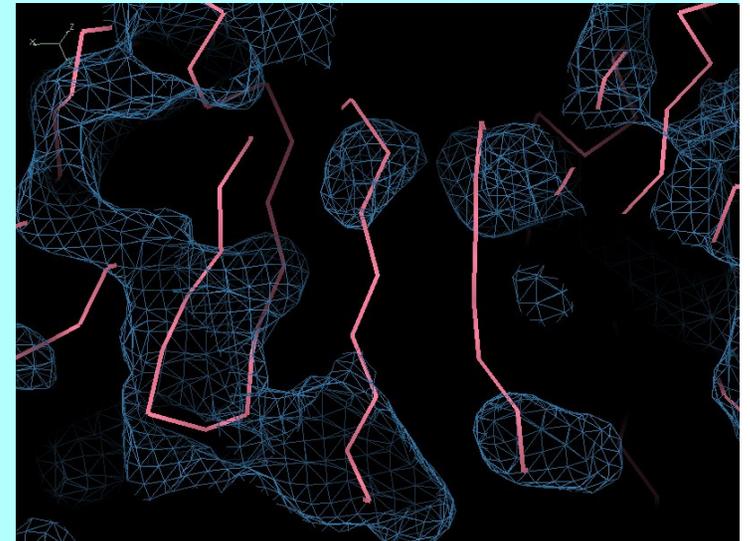
Statistical density modification with cross-crystal averaging

Cell receptor at 3.5/3.7 Å. Data courtesy of J. Zhu

Crystal 1 (4 copies)

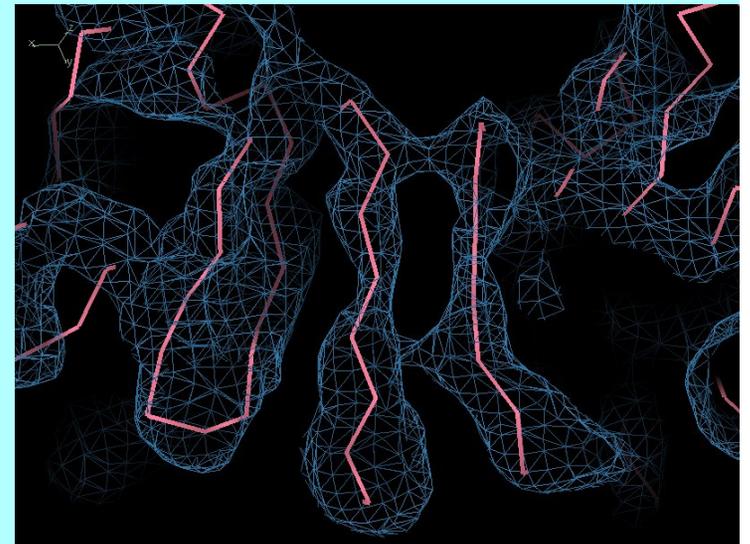
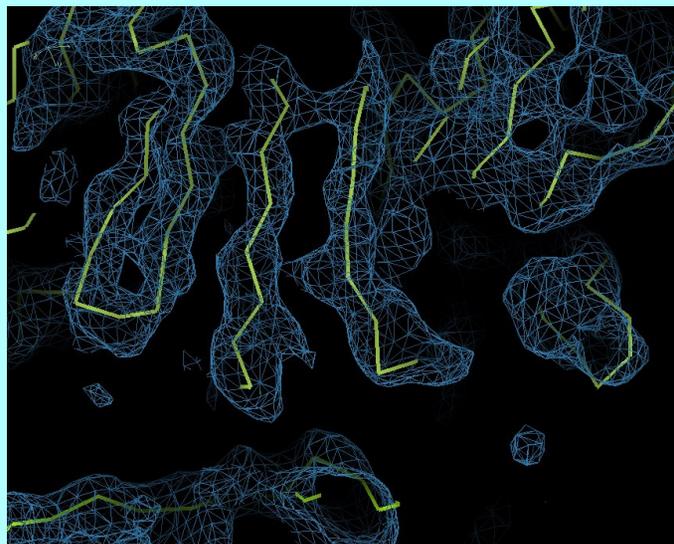


Crystal 2 (2 copies)



RESOLVE
density
modification

PHENIX
Multi-crystal
averaging



The PHENIX Project



Lawrence Berkeley Laboratory

Paul Adams, Ralf Grosse-Kunstleve, Pavel Afonine, Nat Echols, Nigel Moriarty, Jeff Headd, Nicholas Sauter, Peter Zwart



Los Alamos National Laboratory

Tom Terwilliger, Li-Wei Hung



Randy Read, Airlie McCoy, Gabor Bunkoczi, Rob Oeffner

Cambridge University



Duke University

Jane & David Richardson, Vincent Chen, Chris Williams, Bryan Arendall, Laura Murray



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