

Whose Fault is it?

Tamás Ungár

Department of Materials Physics, Eötvös University Budapest, Hungary

The Power of Powder Diffraction

Erice-2011, 2 - 12 June, Erice, Italy

The Effect of Faulting and Twinning on the Profiles of Diffraction Peaks

Tamás Ungár

Department of Materials Physics, Eötvös University Budapest, Hungary

The Power of Powder Diffraction

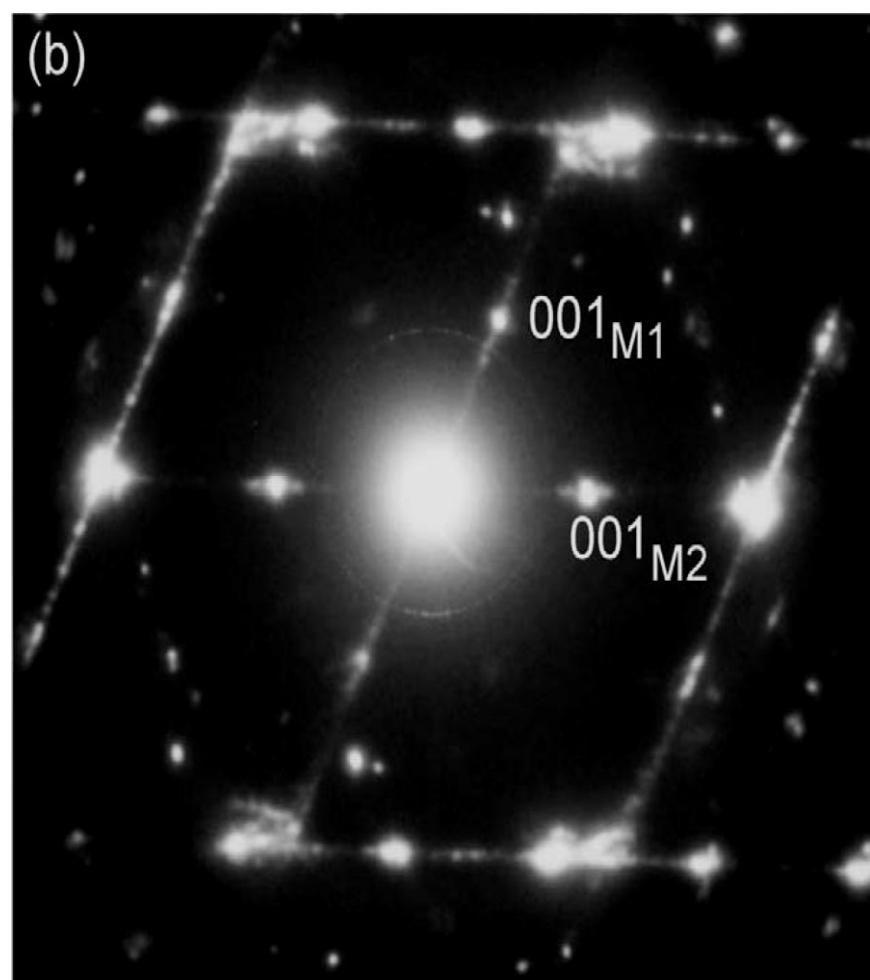
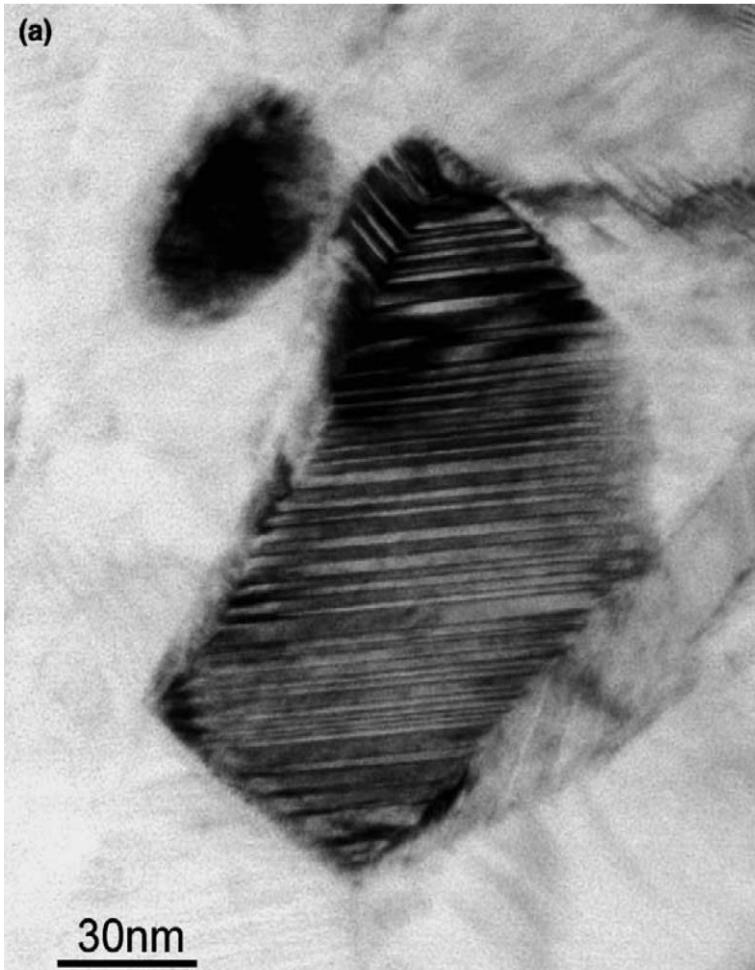
Erice-2011, 2 - 12 June, Erice, Italy

Twinning in NiTi nanograins

T. Waitz, V. Kazykhanov, H.P. Karnthaler, *Acta Materialia* 52 (2004) 137–147

Twinned nanocrystalline grains

streaking is typical for faults



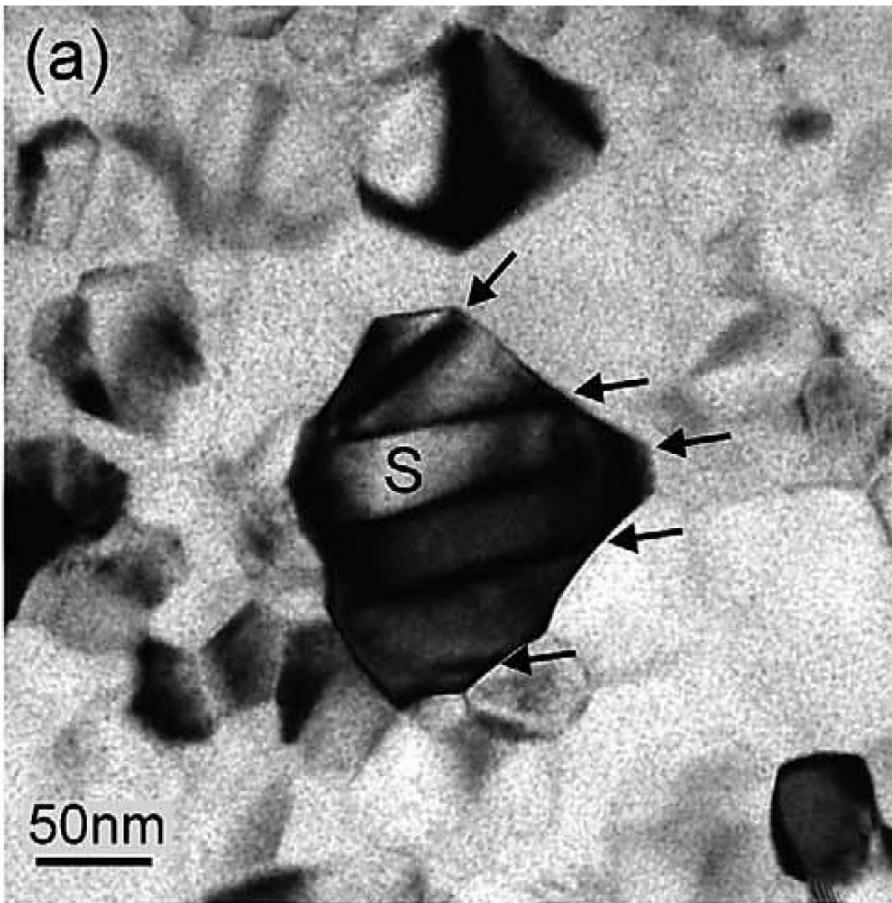
T. Waitz, V. Kazykhanov, H.P. Karnthaler:

Martensitic phase transformations in nanocrystalline NiTi studied by TEM

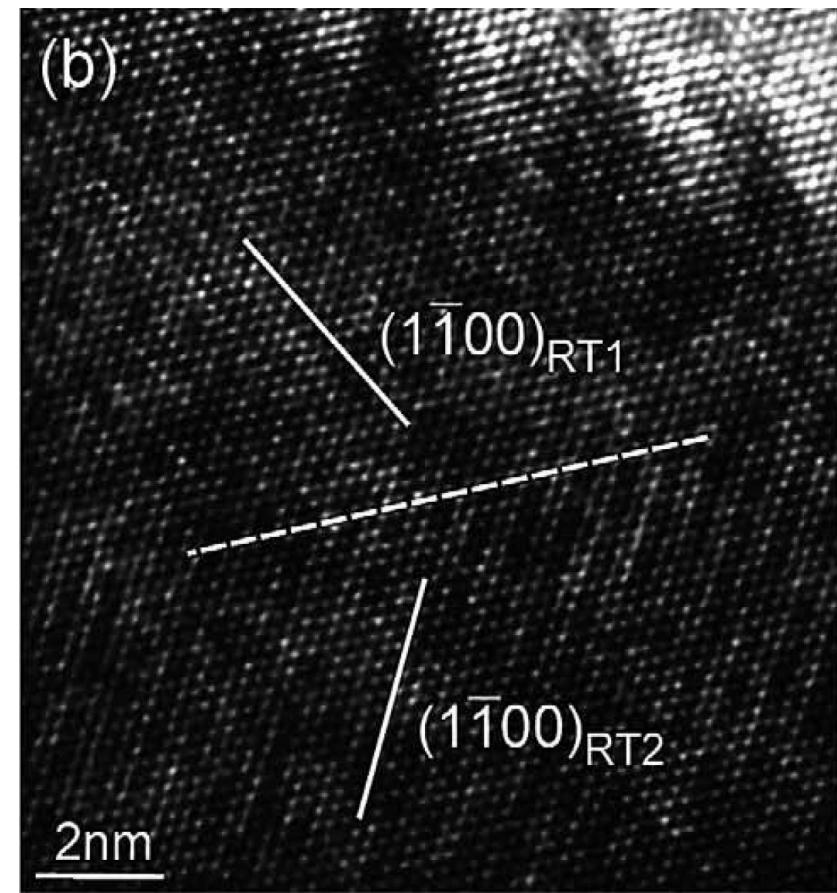
Acta Materialia 52 (2004) 137–147

Perfect matching along the twin plane

Bright field image of a **twinned grain**.



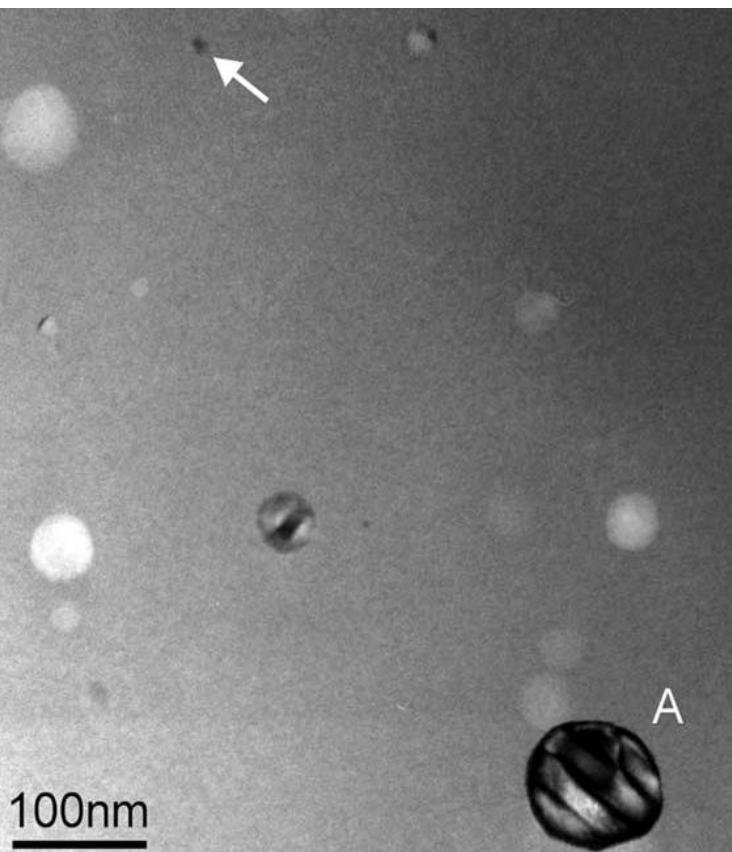
HRTEM image of a grain containing two **twin related variants**: RT1 & RT2



Twinning in NiTi nanograins

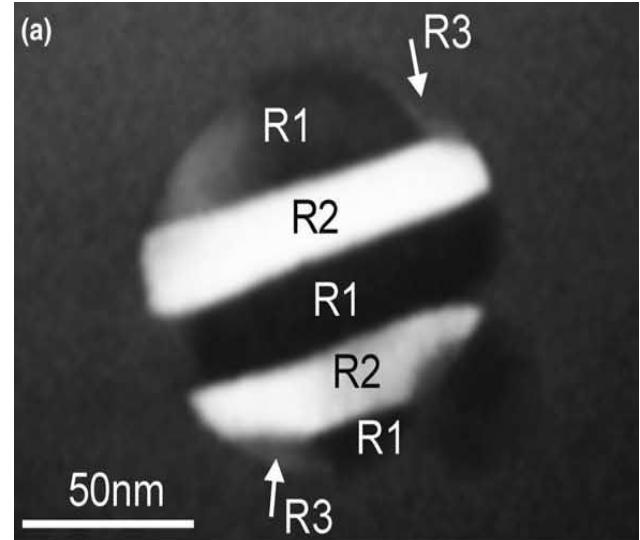
T. Waitz, H.P. Karnthaler, *Acta Materialia* 52 (2004) 5461–5469

Devitrified nanocrystalline particles

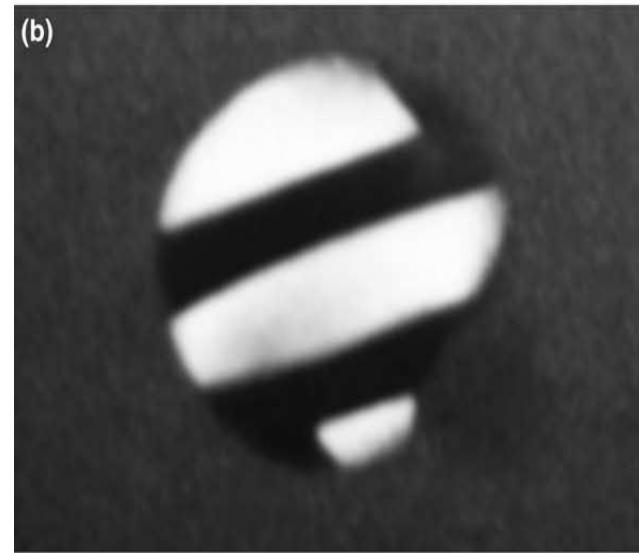


twin related
R1-R2-R3
phases

Dark-Field
with **R2** reflection

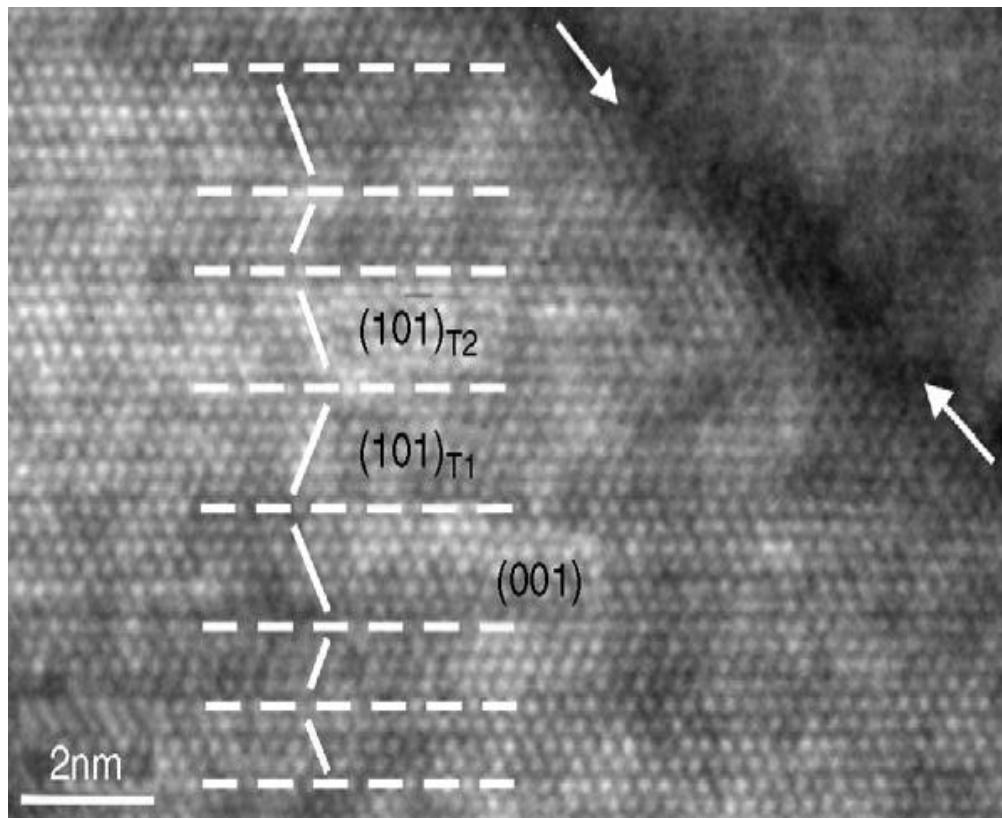
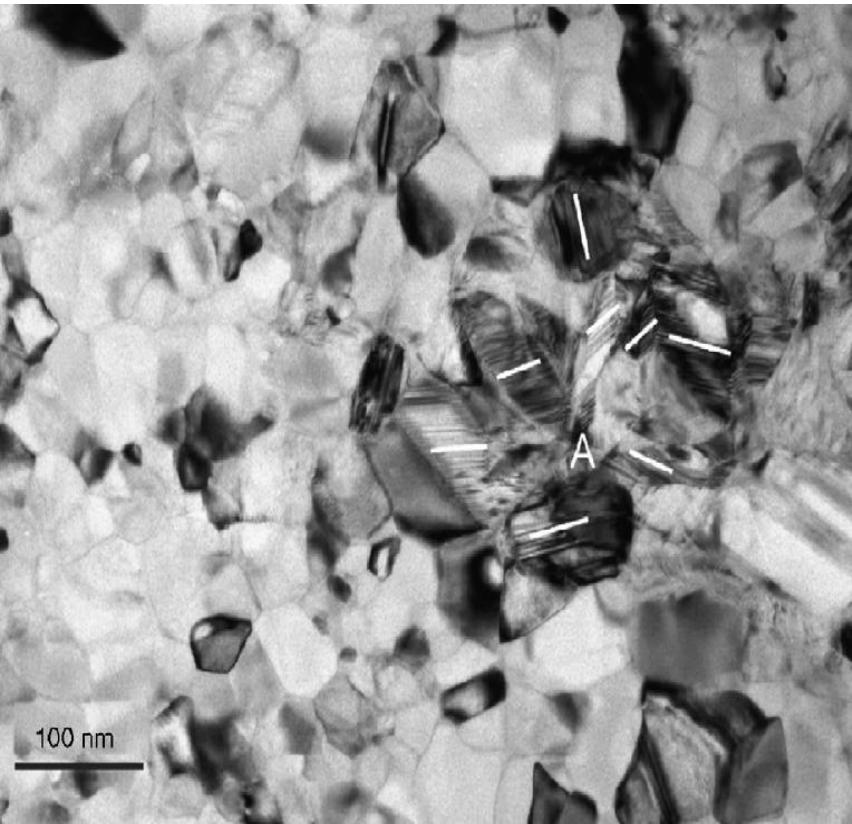


Dark-Field
with **R1** reflection



Ni-Ti shape-memory alloys deform by twinning

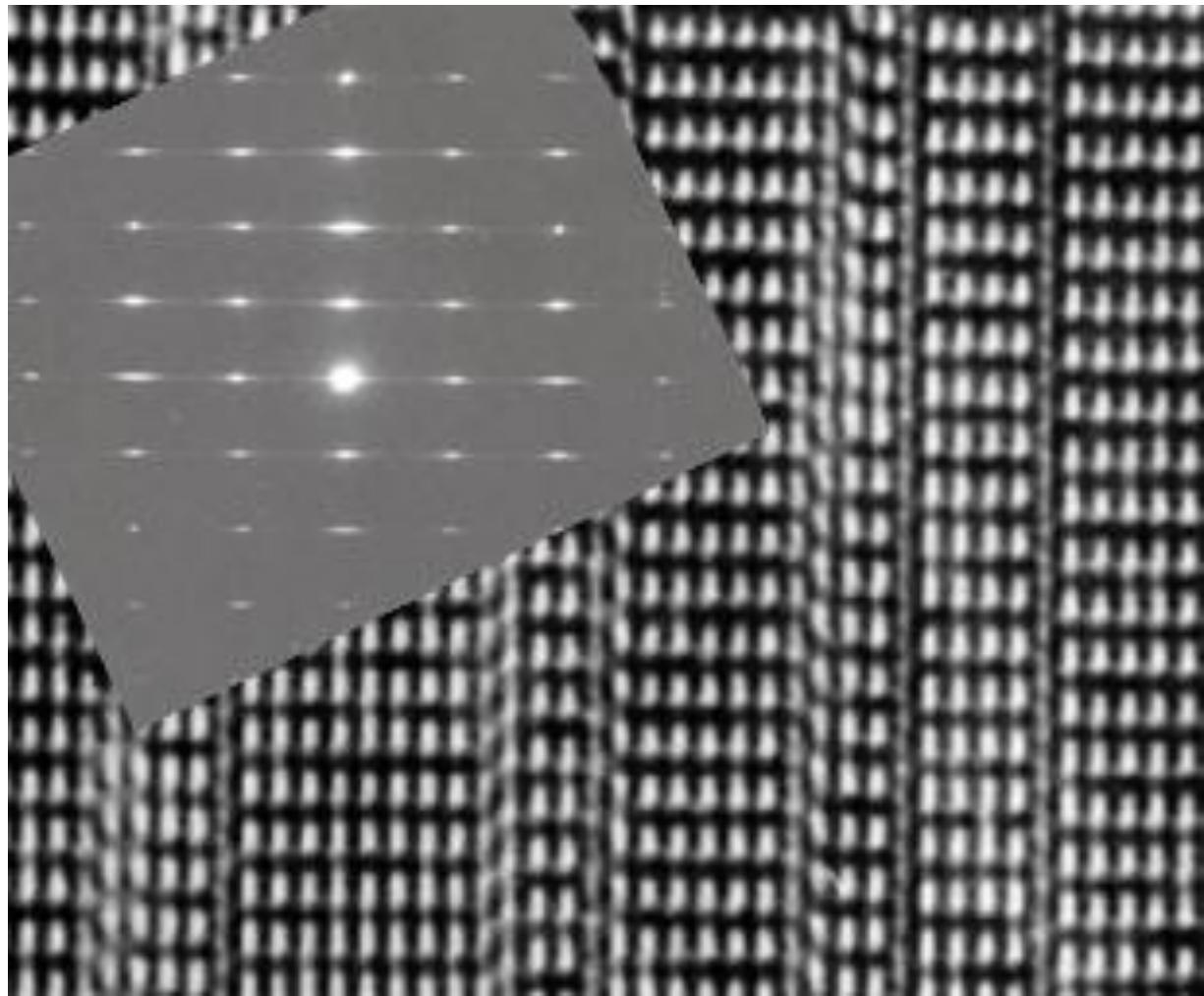
T. Waitz, T. Antretter, F.D. Fischer, N.K. Simha, H.P. Karnthaler: *J. Mech. Phys. Solids*, 55 (2007) 419–444



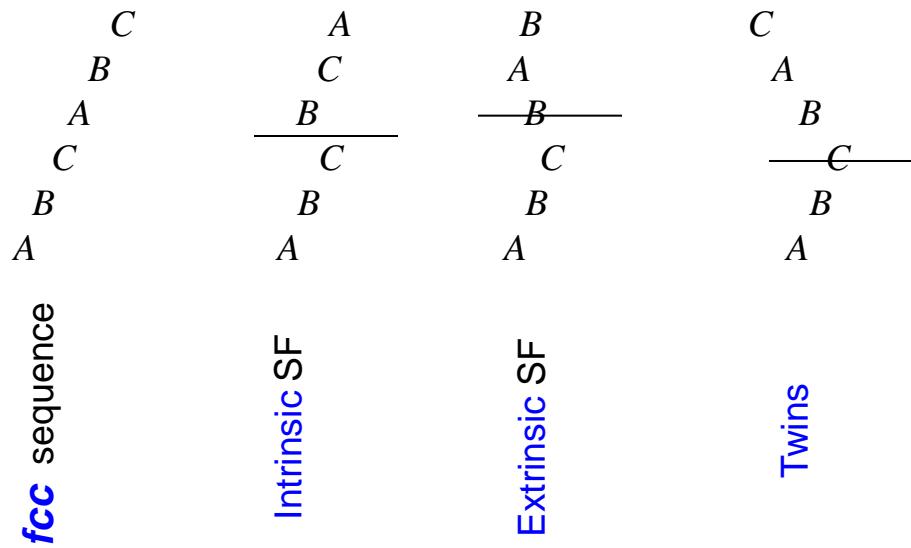
Planar Defects in Magnesium-Fluorogermanate

P. Kunzmann in J.M. Cowley, Acta Cryst. (1976). A32, 83

Streaking normal to the fault planes



Faulting along the close-packed planes in *fcc* or *hcp* crystals



What about *hcp* metals like:

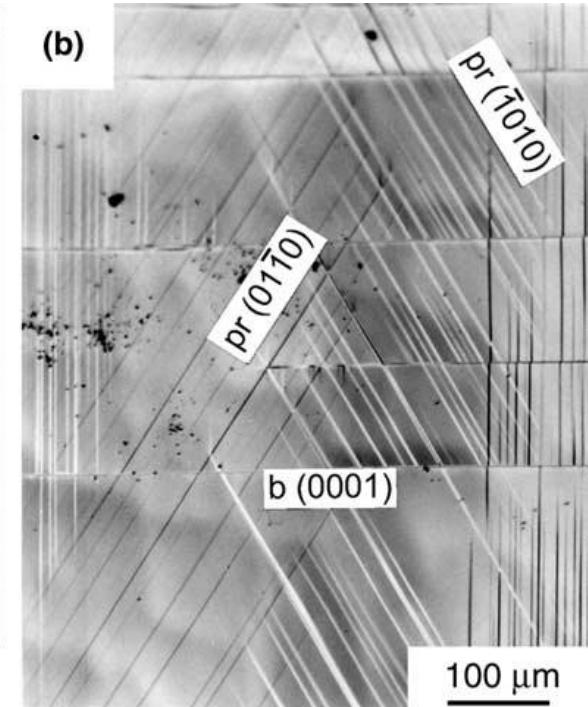
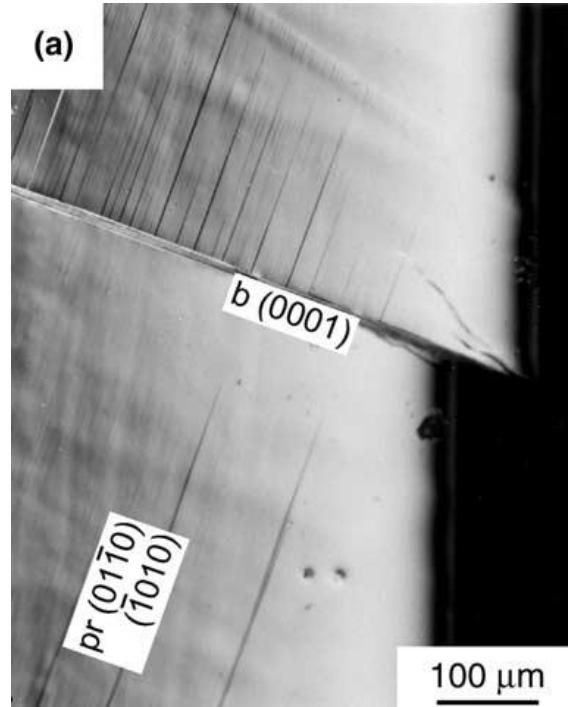
Mg, Ti, Zr, Be, Zn ?

and their alloys

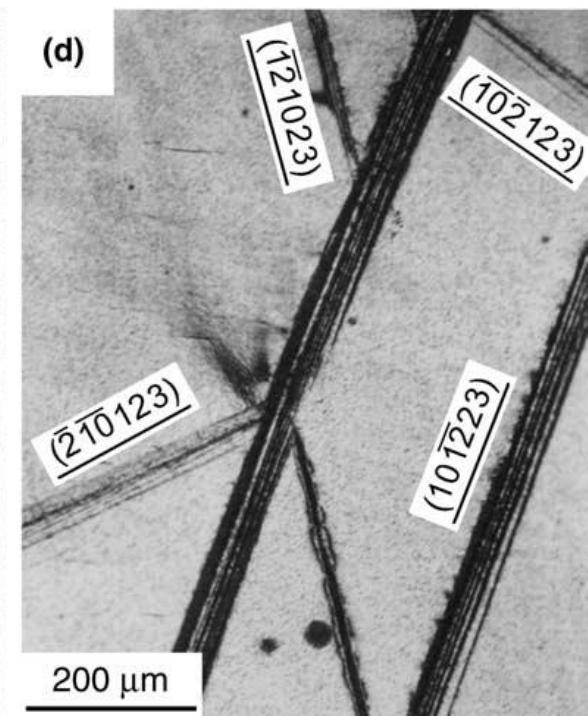
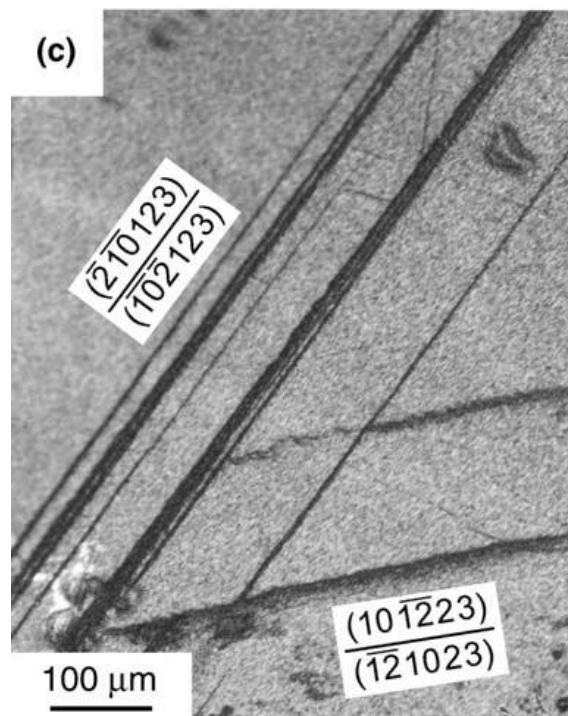
Ti-Al *hcp*-DO₁₉

K. Kishida, Y. Takahama, H. Inui,
Acta Mater., 52 (2004) 4941-4952

prismatic
planes



pyramidal
planes

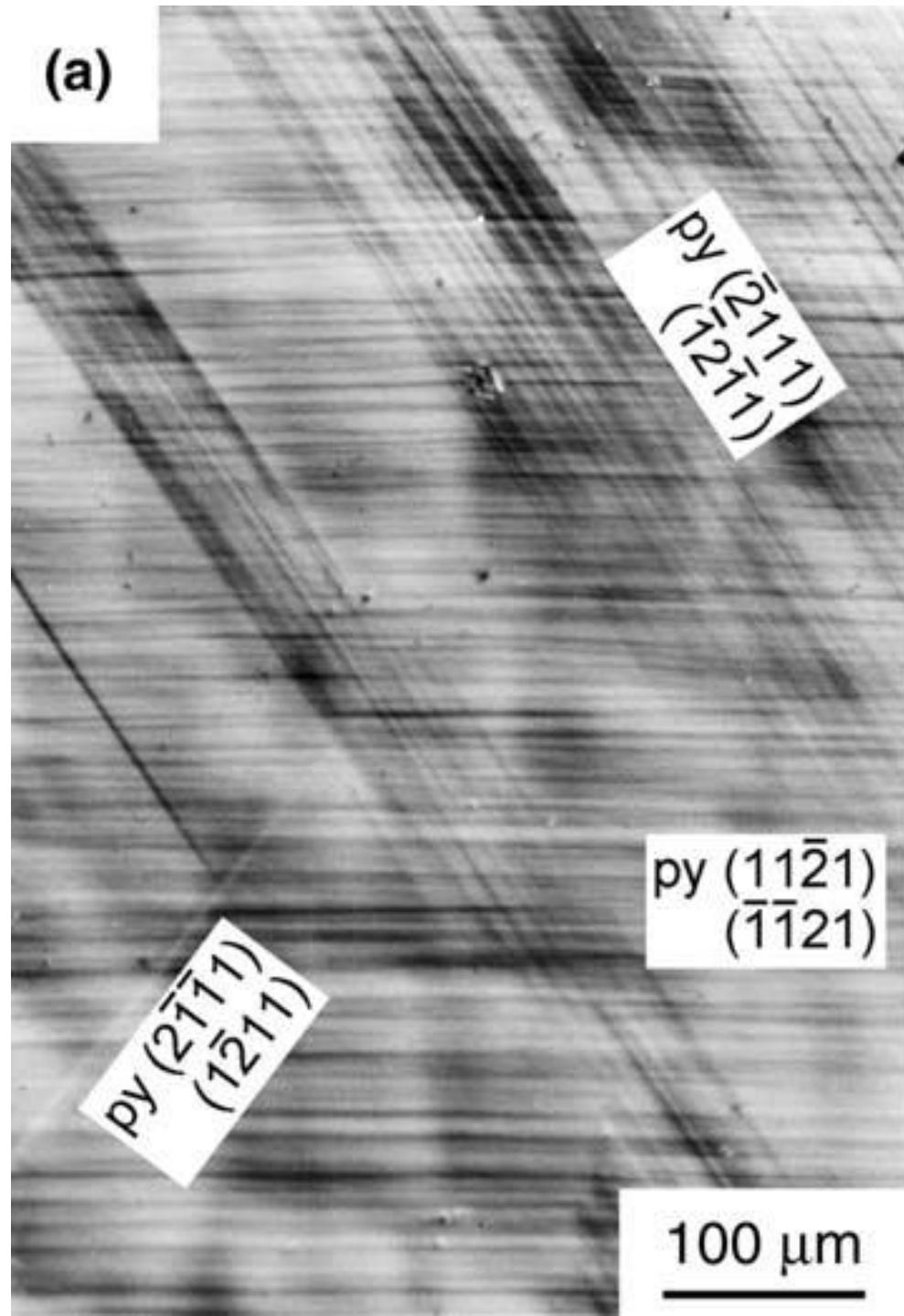


optical micrographs

Ti-Al *hcp*-DO₁₉

K. Kishida, Y. Takahama, H. Inui,
Acta Mater., 52 (2004) 4941-4952

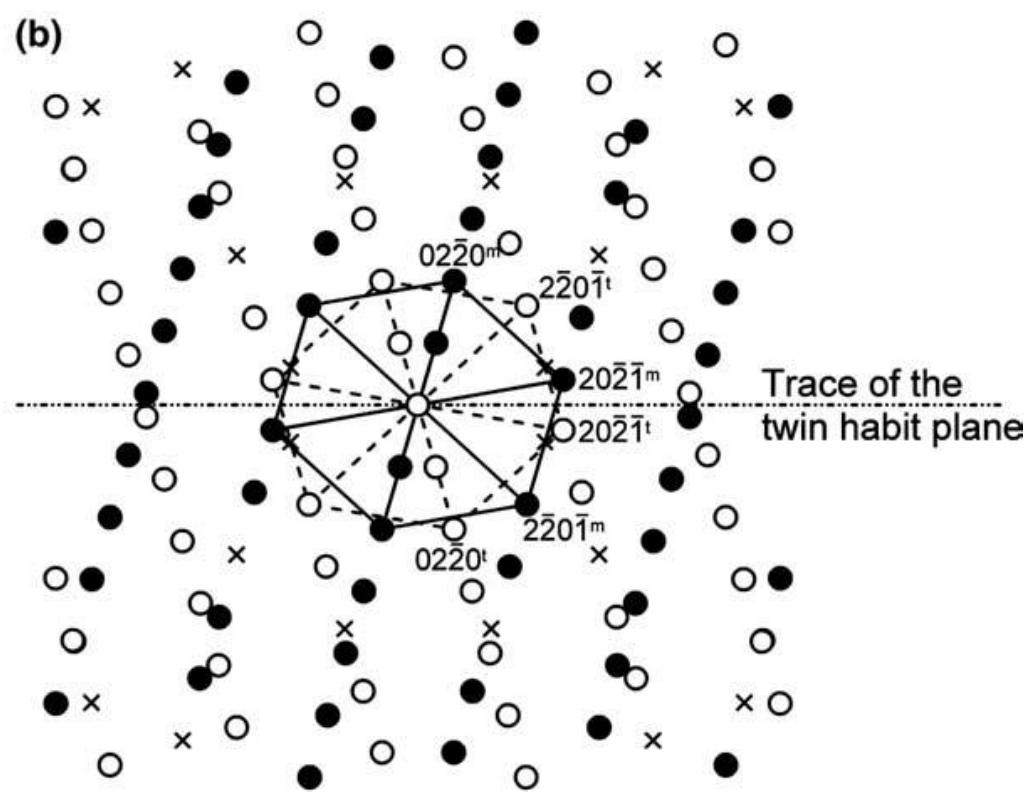
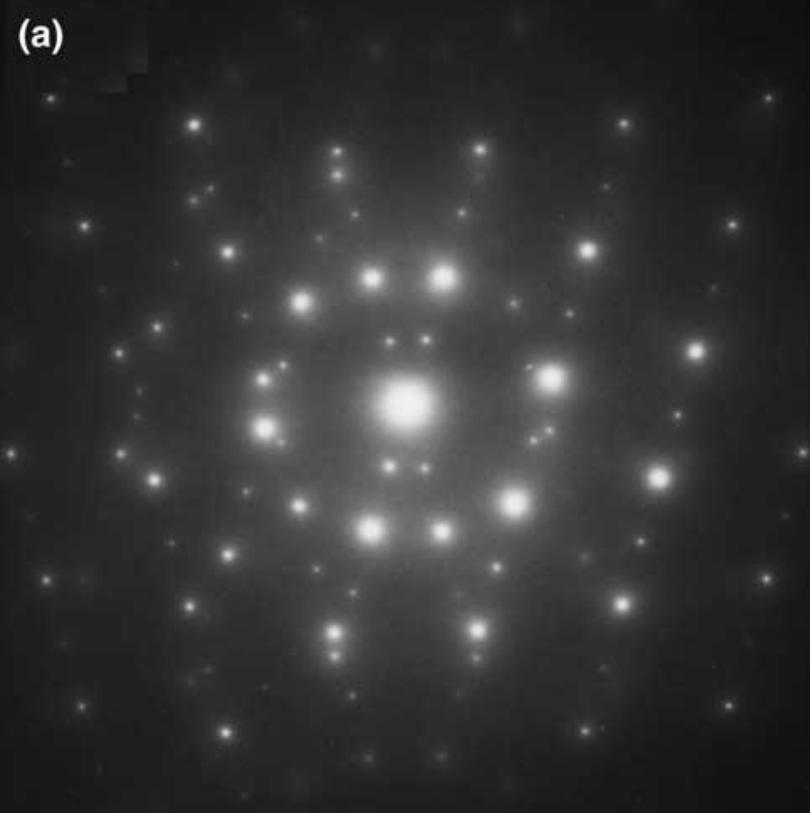
pyramidal planes



Ti-Al *hcp*-DO₁₉

K. Kishida, Y. Takahama, H. Inui,
Acta Mater., 52 (2004) 4941-4952

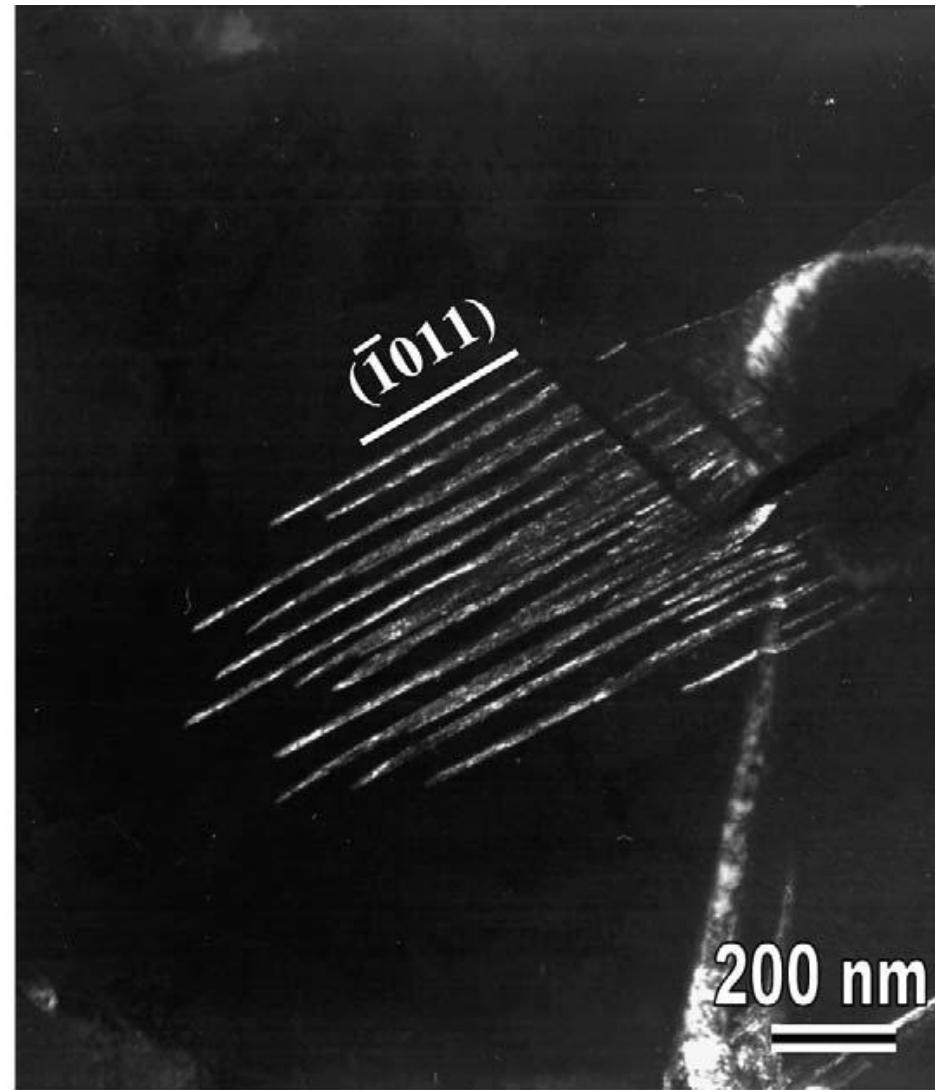
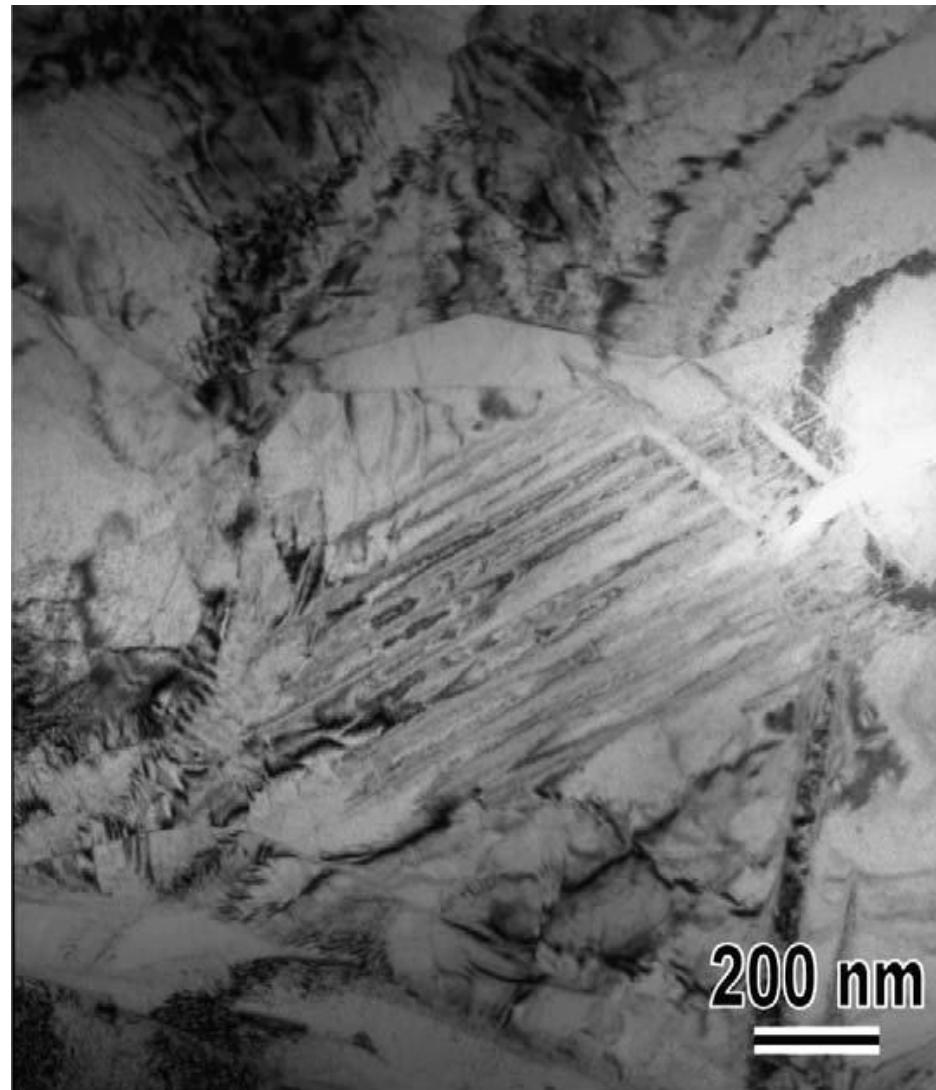
Diffraction spots of
mother and twin crystal
never coincide:
non-merohedral



Ti-6Al-4V

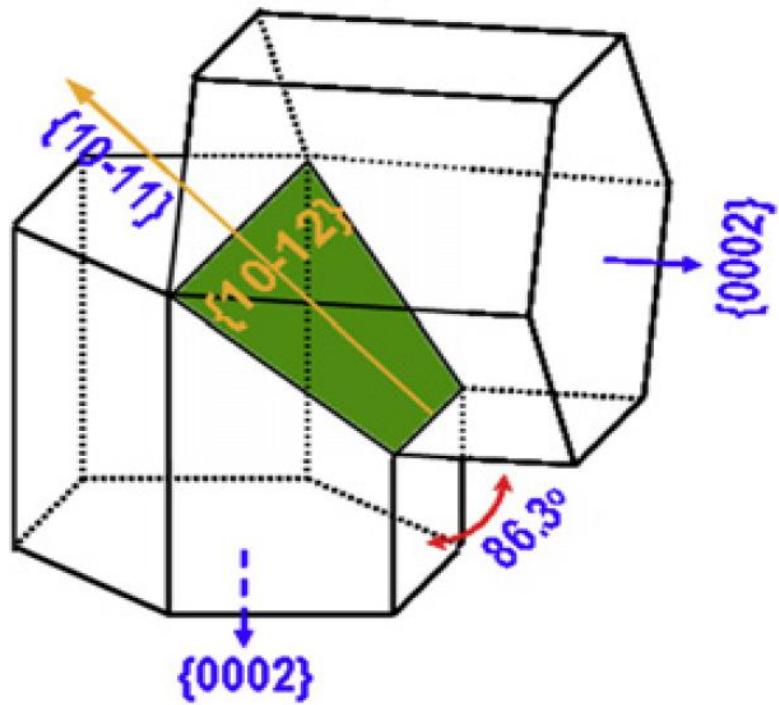
G.G.Yapici, I.Karaman, Z.P.Luo, Acta Mater, 54 (2006) 3755

Deformation twins along the $\bar{1}0\cdot1$ pyramidal plane

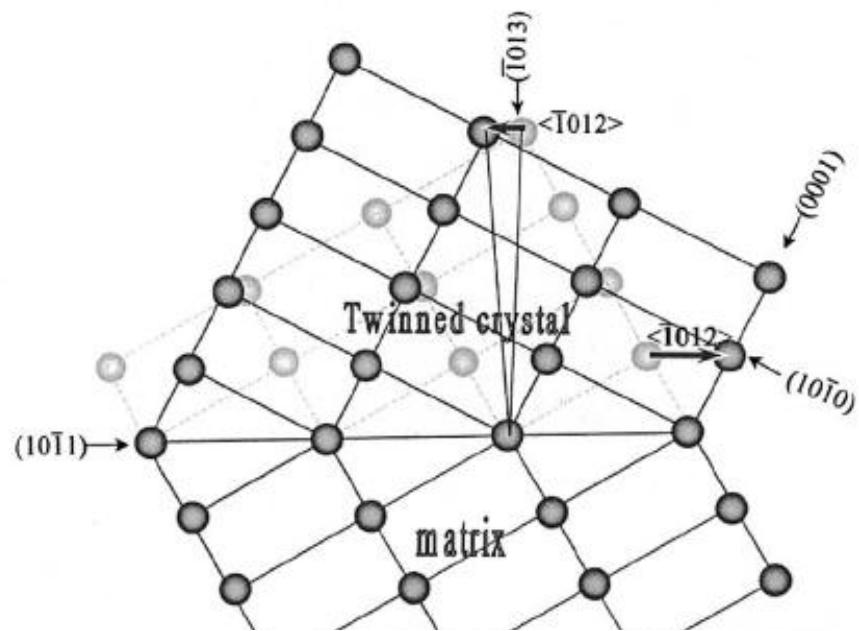


Twinning on pyramidal planes in *hcp* crystals

{10.2}



{10.1}



non-merohedral



L. Wu, A. Jain, D.W. Brown, G.M. Stoica,
S.R. Agnew, B. Clausen, D.E. Fielden,
P.K. Liaw Acta Materialia 56 (2008) 688–695

I. Kim, W.S. Jeong, J. Kim, K.T. Park,
D.H. Shin Scripta Materialia 45 (2001) 575-581

Philosophy of the present line-profile analysis

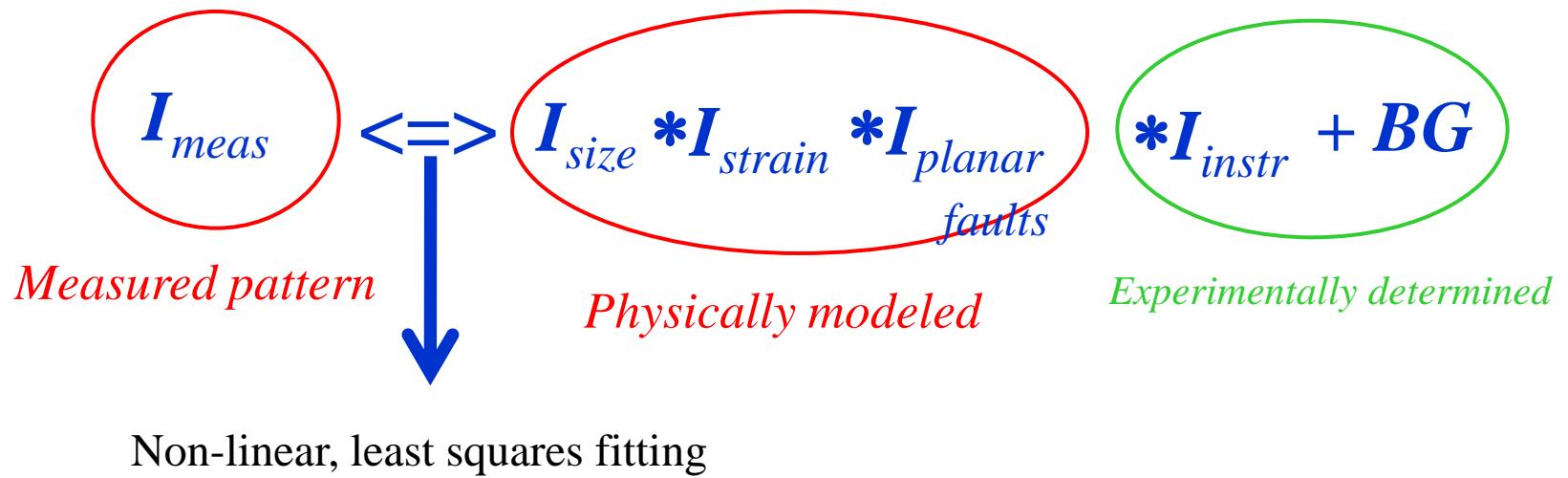
Bottom-up approach

profile-functions are
created by **theoretical methods**
based on **real lattice defects**

diffraction patterns are **fitted** by
patterns constructed by using

defect-related profile-functions

Philosophy of line profile analysis



$$I_{Strain} : \rho, M, q$$

$$M = R_e \rho^{1/2}$$

$$I_{Size} : m, \sigma$$

$$I_{planar} : \alpha \text{ or } \beta$$

Strain-broadening: strain-profile

is given by:

$$\langle \varepsilon_{g,L}^2 \rangle$$

mean-square-strain

where the strain Fourier coefficients are:

$$A^D(L) = \exp\{-2\pi^2 g^2 L^2 \langle \varepsilon_{g,L}^2 \rangle\}$$

Dislocation-model for $\langle \varepsilon_{g,L}^2 \rangle$: Krivoglaz-Wilkens [1970]:

$$\langle \varepsilon_{L,g}^2 \rangle = \frac{\rho \cdot C \cdot b^2}{4\pi} f(\eta)$$

b : Burgers vector

ρ : dislocation density

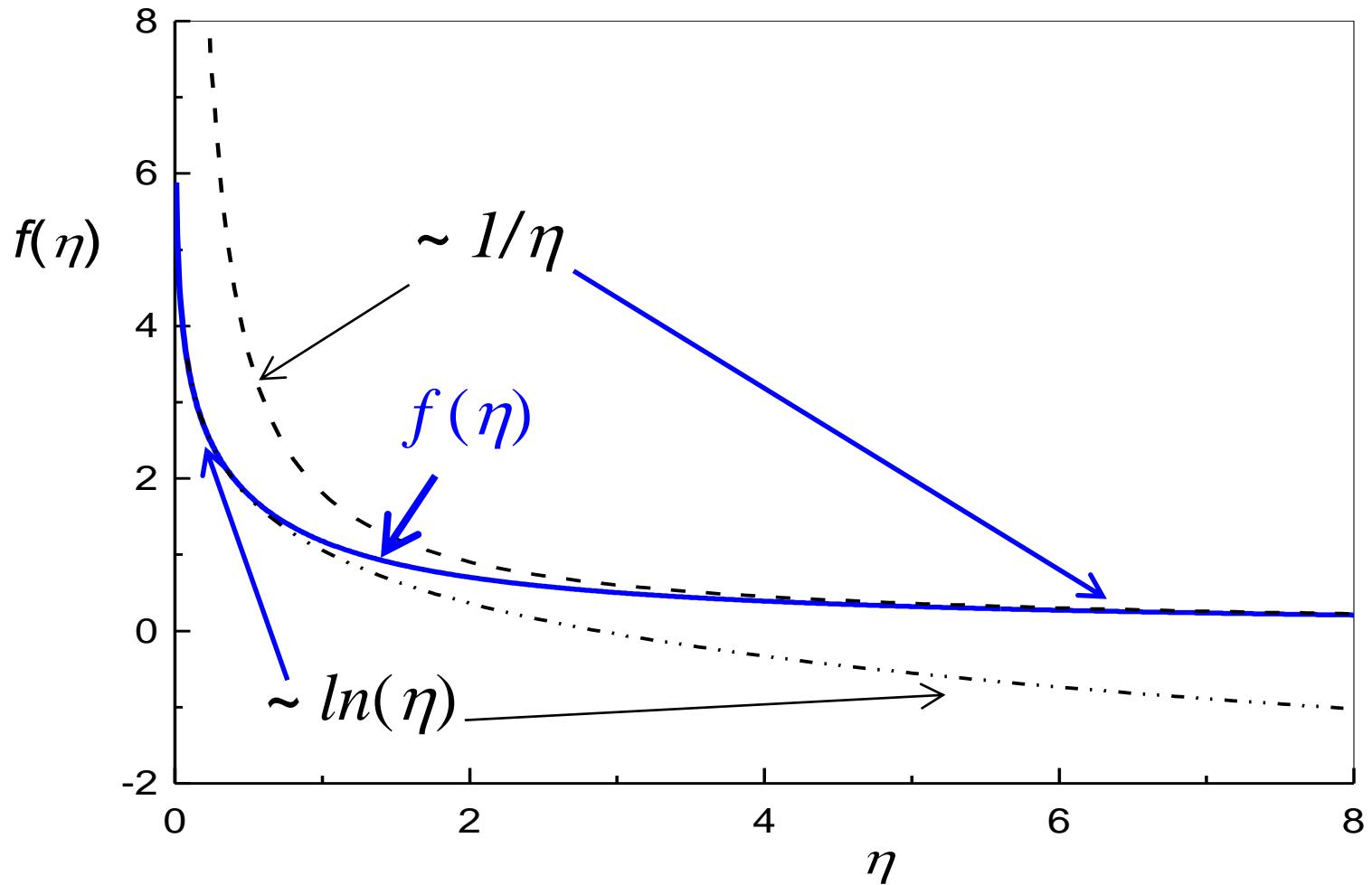
C : Contrast factor of dislocations

$f(\eta)$: Wilkens function [1970]

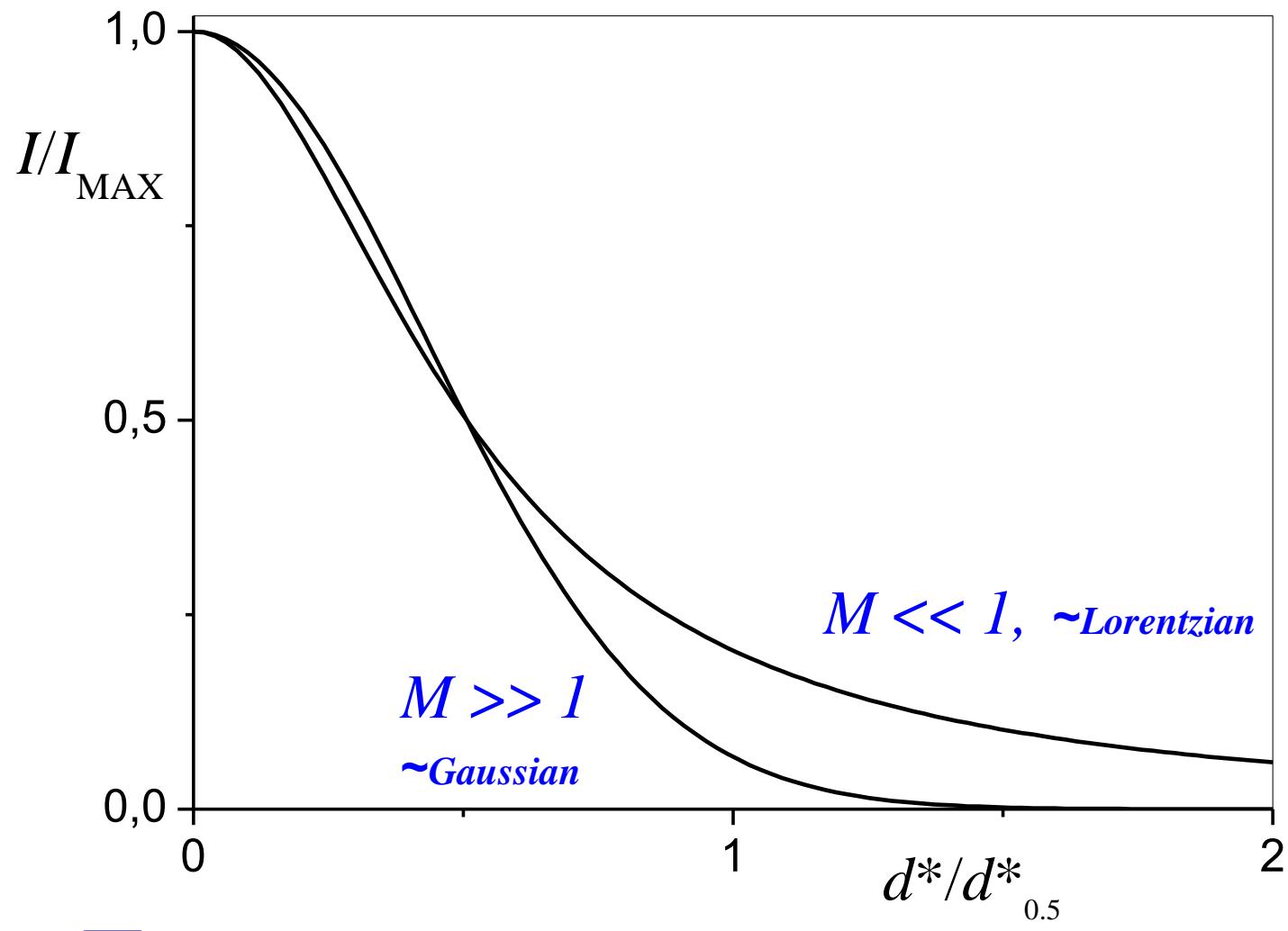
$$\eta = L/R_e$$

The Wilkens function [1970]

$$\eta = L/R_e$$



Strain-profiles normalized



$$M = R_e \sqrt{\rho}$$

Strain profile:

inverse Fourier transform of the *strain Fourier coefficients*

$$I^D(s) = \int \exp\{-2\pi^2 L^2 g^2 \langle \varepsilon_{g,L}^2 \rangle\} \exp(2\pi i L s) dL$$

where:

$$\langle \varepsilon_{g,L}^2 \rangle = (b/2\pi)^2 \pi \rho C f(\eta)$$

Size profile: I^S

assuming **log-normal** size distribution:

$$f(x) = \frac{1}{(2\pi)^{1/2}\sigma} \frac{1}{x} \exp\left\{-\frac{[\log(x/m)]^2}{2\sigma^2}\right\}$$

m : median

σ : variance

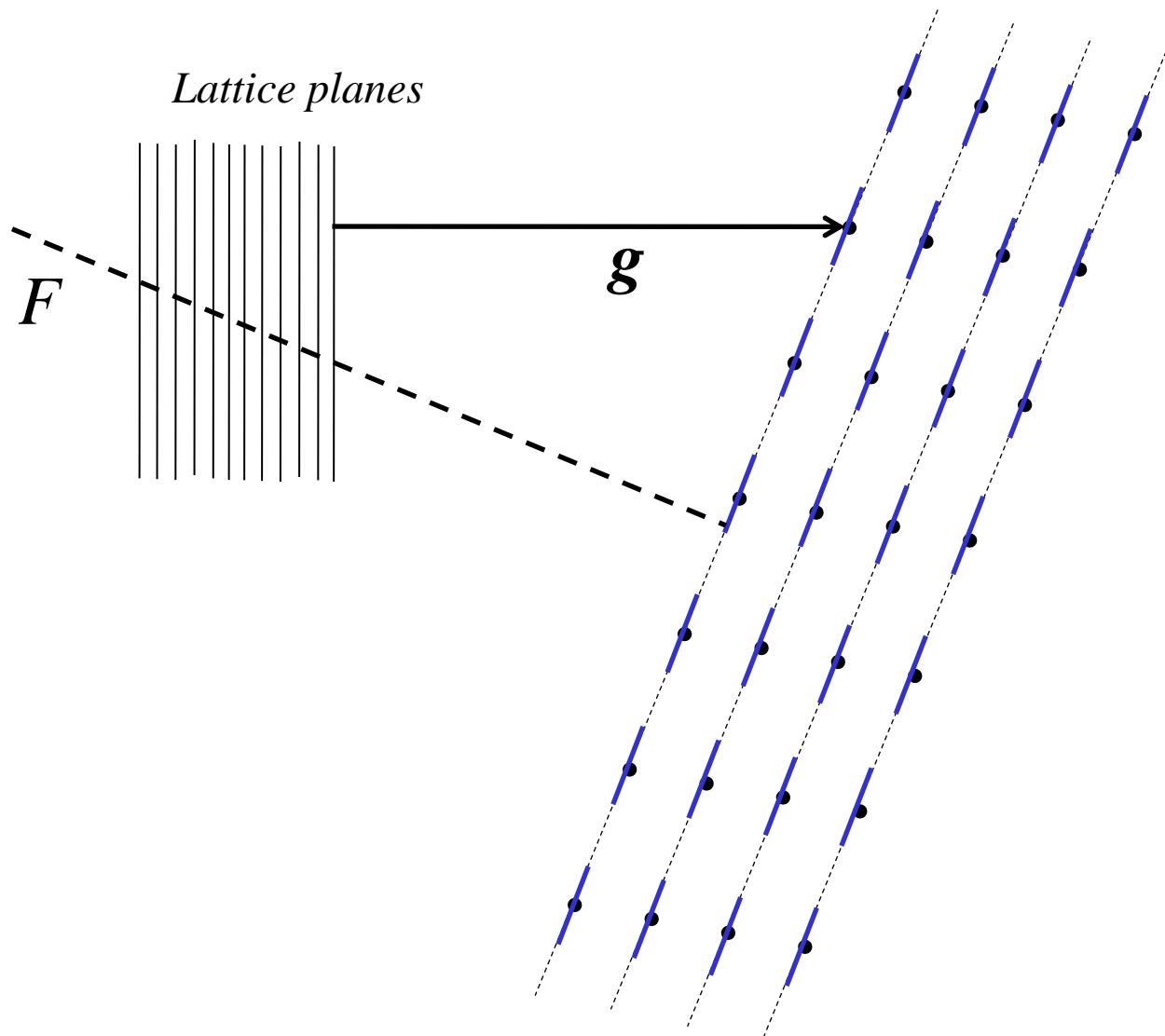
$$I^S(s) = \int_0^\infty \mu \frac{\sin^2(\mu\pi s)}{(\pi s)^2} \operatorname{erfc}\left[\frac{\log(\mu/m)}{2^{1/2}\sigma}\right] d\mu$$

shape-anisotropy can be allowed for

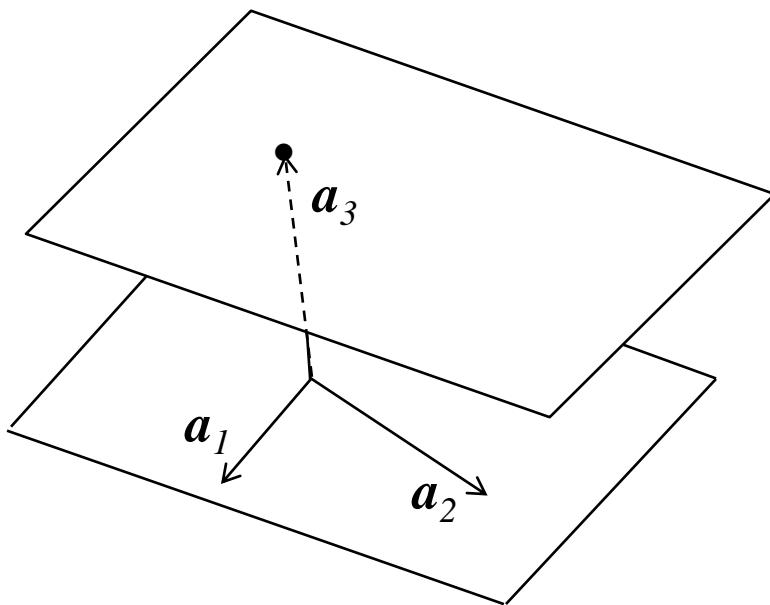
Profile functions for *faulting* and *twinning*

Line broadening from streaking

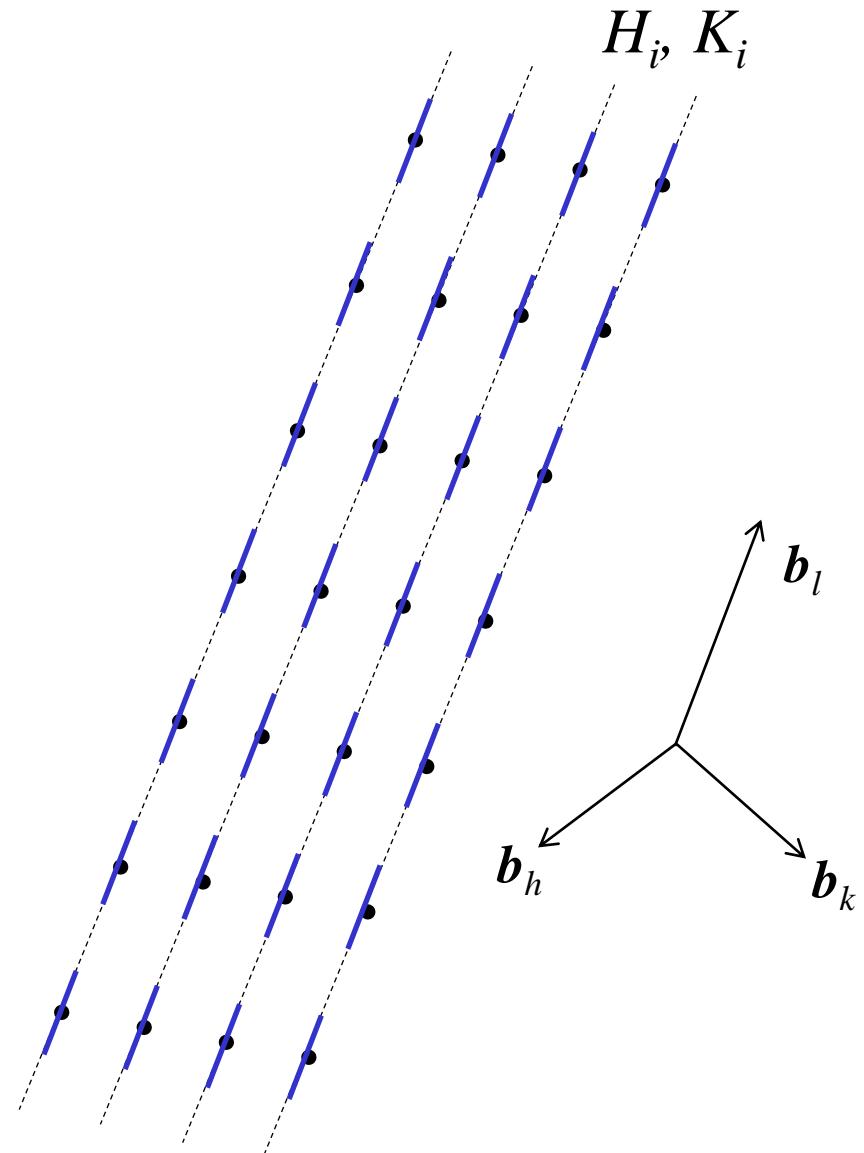
Lattice planes



Coordinate systems



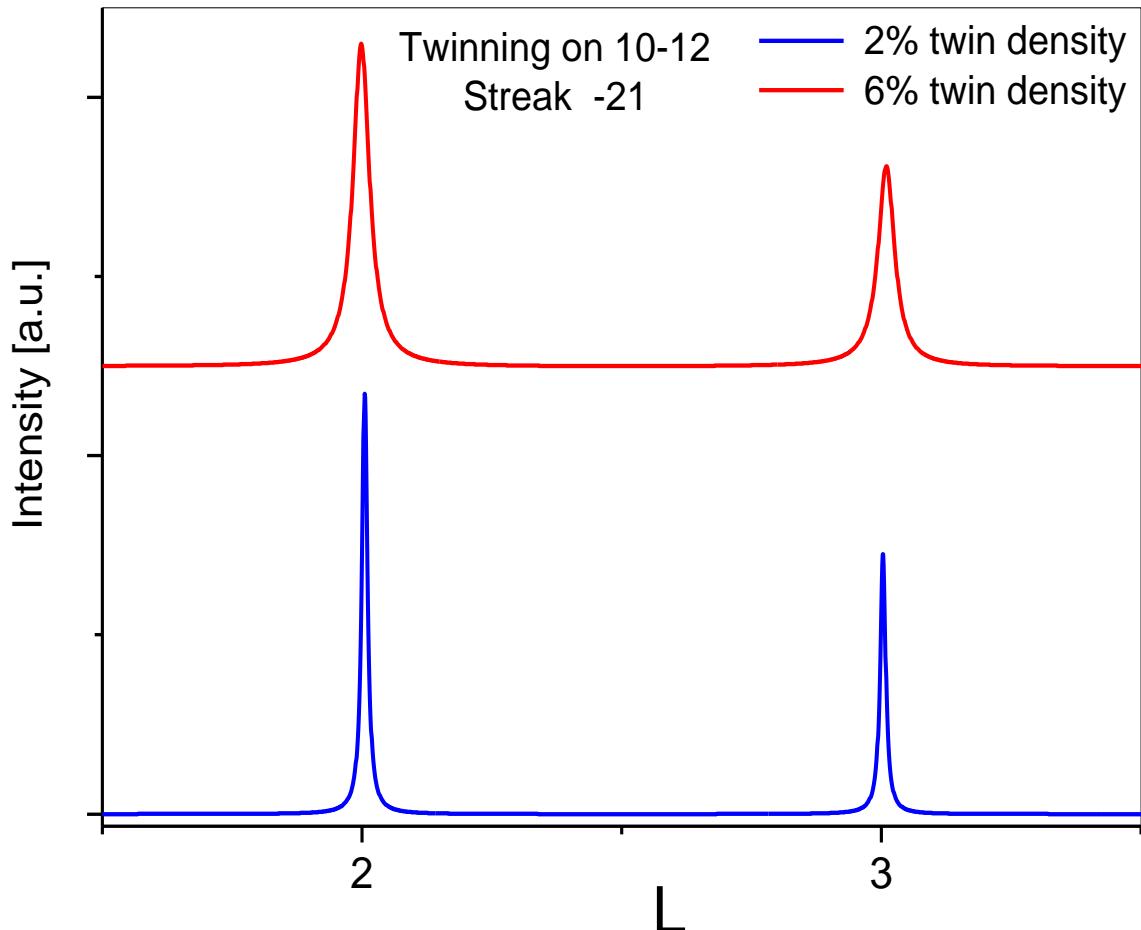
tricline



Intensity distribution along the $(H_i, K_i) = (-2 \ 1)$ streaks

DIFFaX, Treacy MMJ, Newsam JM, Deem MW, Proc. Roy. Soc. London A, (1991) 433, 499-520

twinning on
pyramidal planes

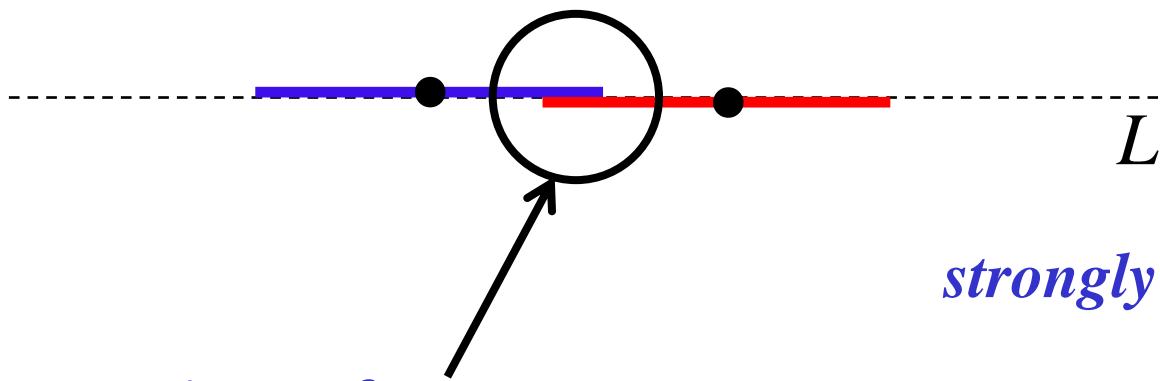


NOT line-profiles yet

Overlapping peaks along the *same streak*: *asymmetry*



almost symmetric peaks

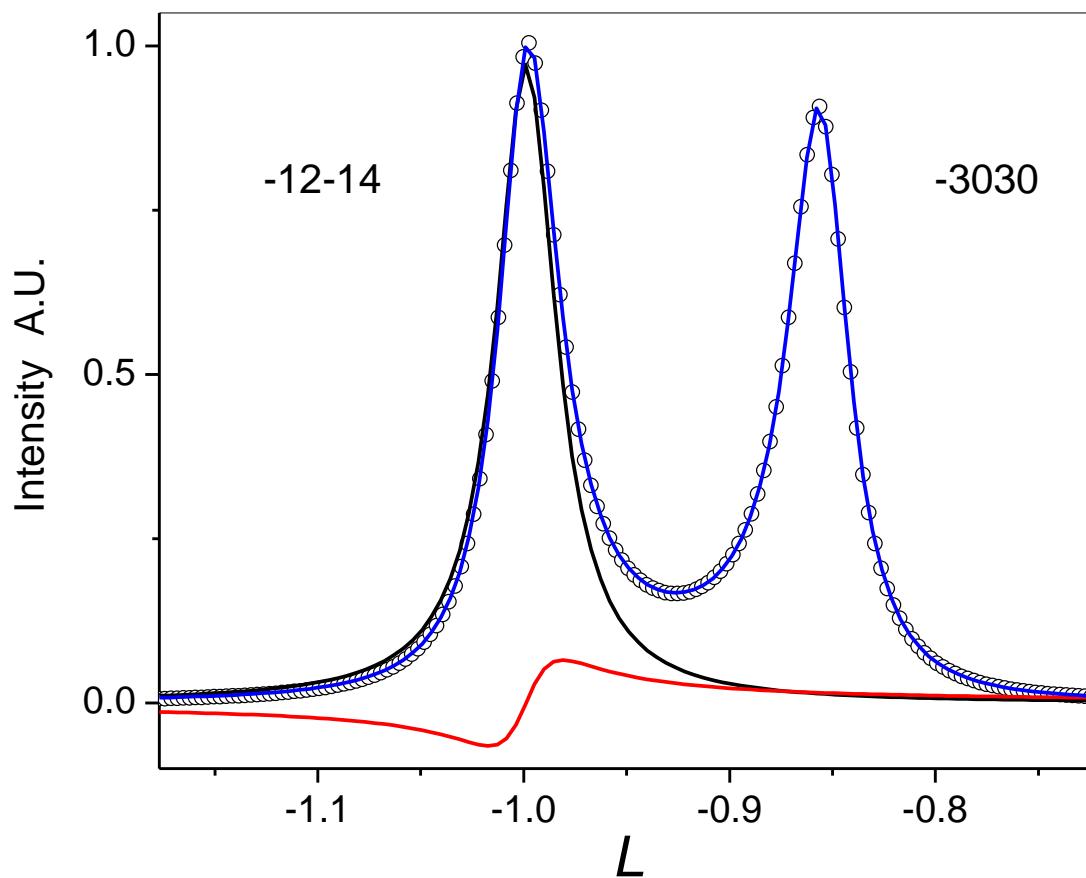


strongly asymmetric peaks

interference

Ti: *pyramidal* twin plane: $11\bar{2}2$

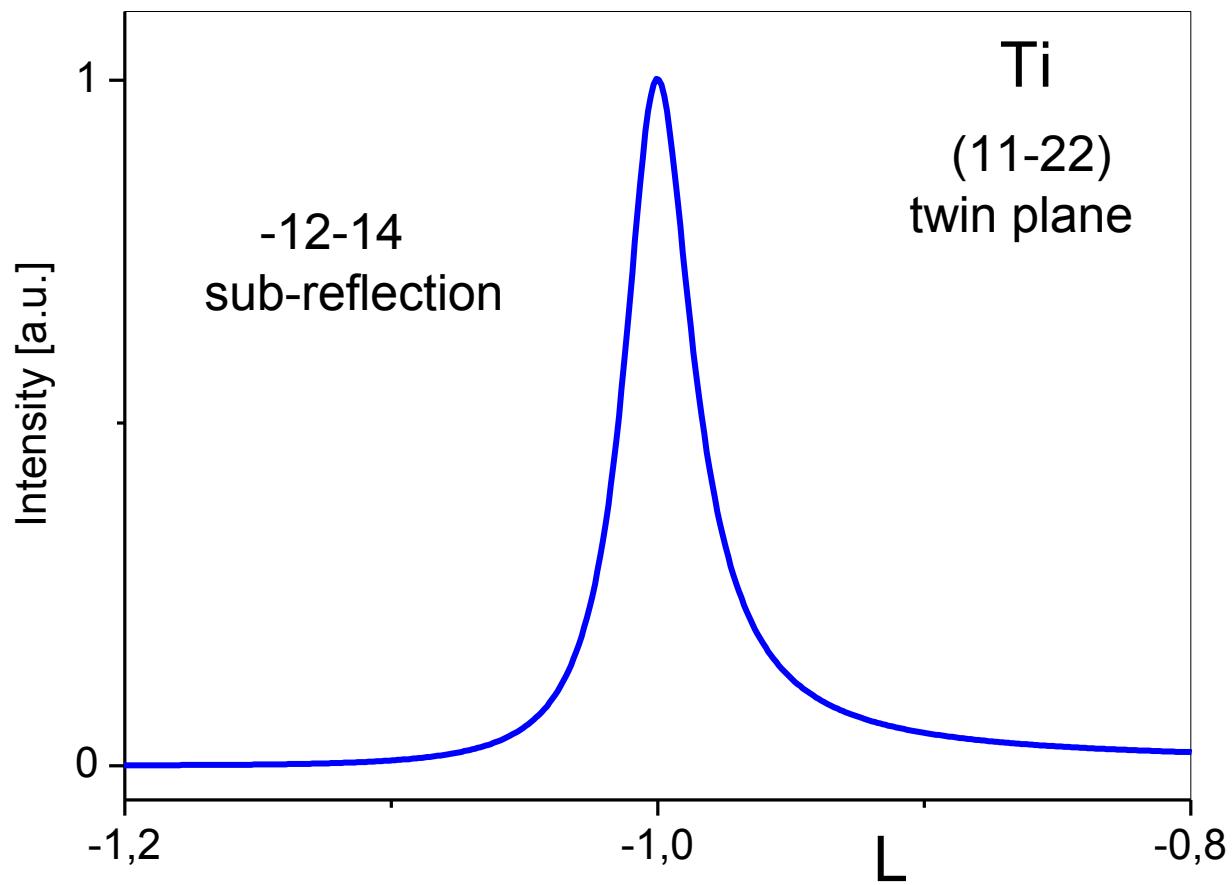
$\{11.4\}$ reflection: $\bar{1}2\bar{1}4$ sub-reflection (parent crystal)
 $\{31.0\}$ reflection: $\bar{3}030$ sub-reflection (twin crystal)



$$\beta = 6 \%$$

Ti: *pyramidal* twin plane: $11\bar{2}2$

only the $\bar{1}2\bar{1}4$ sub-reflection in the $\{11.4\}$ reflection

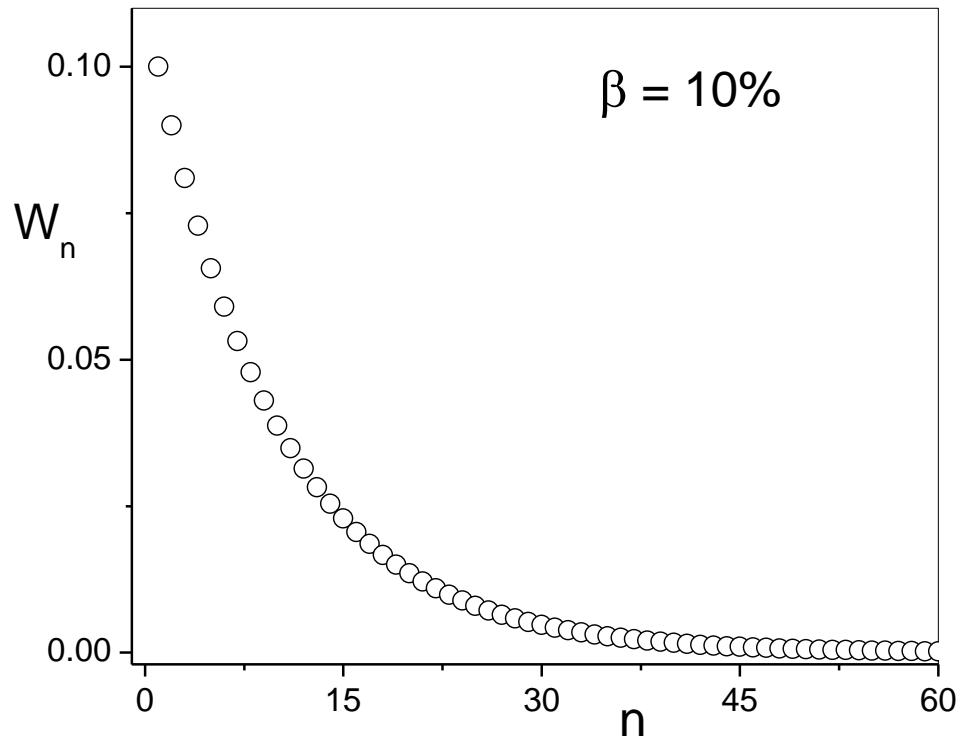


Intensity distribution in the streaks

twinning probability or twin boundary frequency: β

fraction of twin lamellae of thickness of n crystal-layers: W_n

$$W_n = \beta(1 - \beta)^{n-1}$$



Intensity distribution in the streaks: $I(L)$

$$I(L) = \frac{I_0}{1 + \frac{4(L - L_0)^2}{FWHM_L^2}} \left[1 + A_{asym} \cdot (L - L_0) \right]$$

where: $FWHM_L = \frac{2\beta}{D_{tri}\sqrt{1-\beta}}$

$I(L)$: *Lorentzian* type function

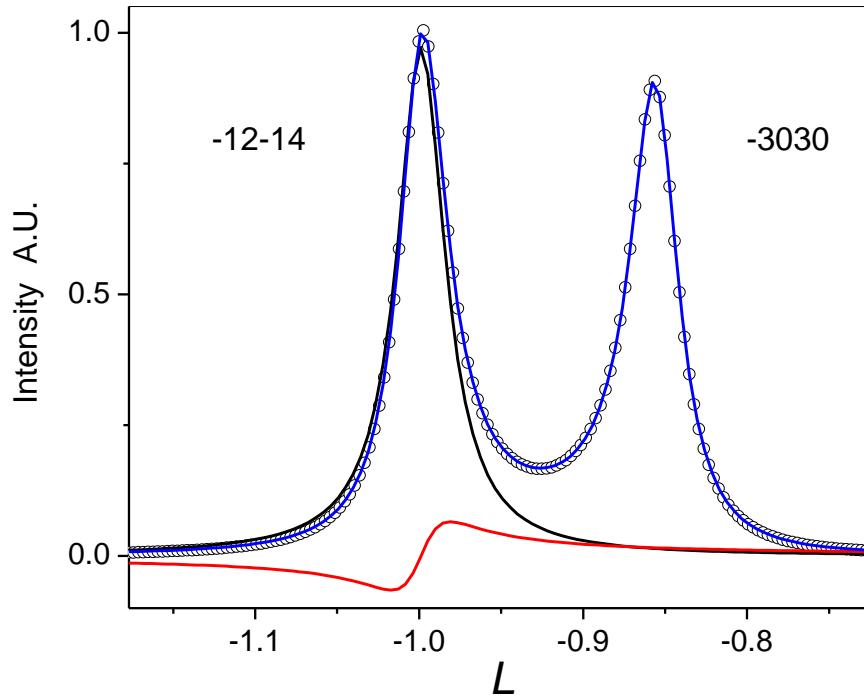
It can been shown that:

$$FWHM_L = \frac{2\beta}{D_{tri}\sqrt{1-\beta}}$$

is a ***universal*** relation for ***twinning***,

where ***D_{tri} = 5.775***

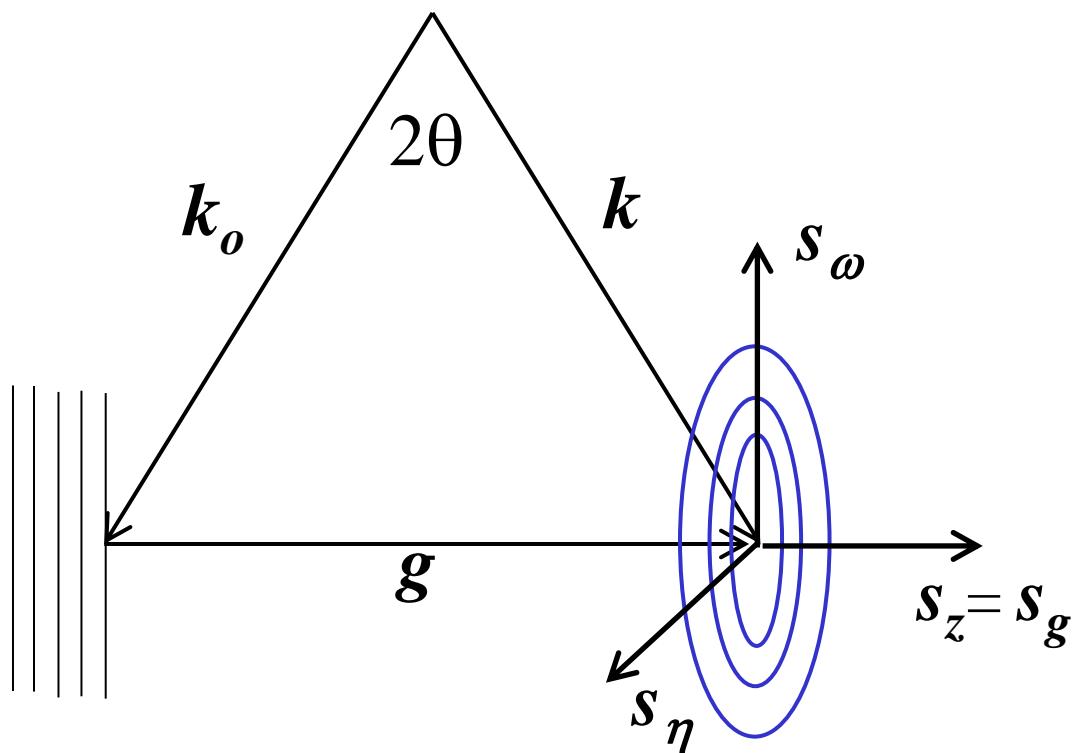
Intensity distribution in the streaks: *sub-profiles*



$$I(L) = \frac{I_0}{1 + \frac{4(L - L_0)^2}{FWHM_L^2}} [1 + A_{asym} \cdot (L - L_0)]$$

$I(L)$: the sum of
a *symmetrical* + an *antisymmetrical*
Lorentzian type function

Intensity distribution in reciprocal-space is 3 dimensional



$$I=I(s_\omega, s_\eta, s_g)$$

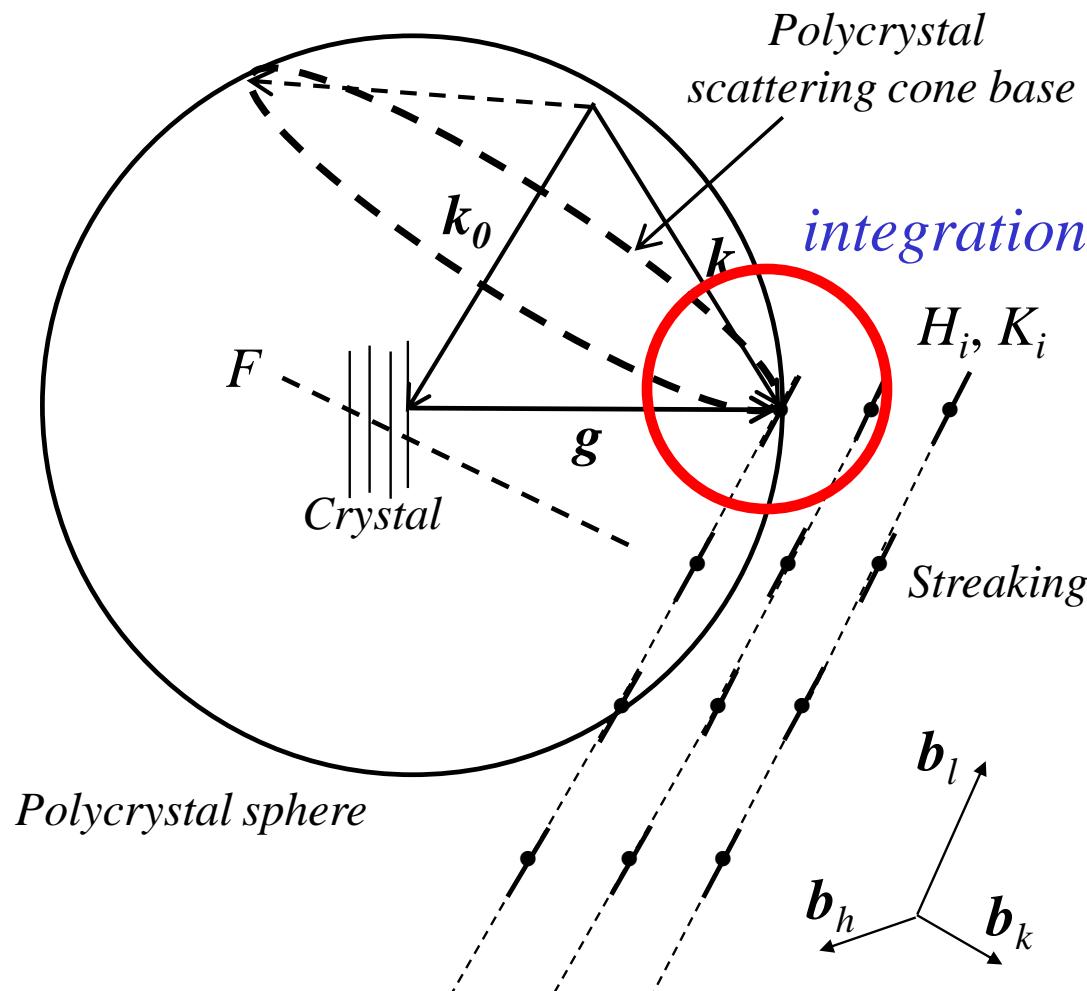
Line profile:

intensity distribution along the diffraction vector: **g**

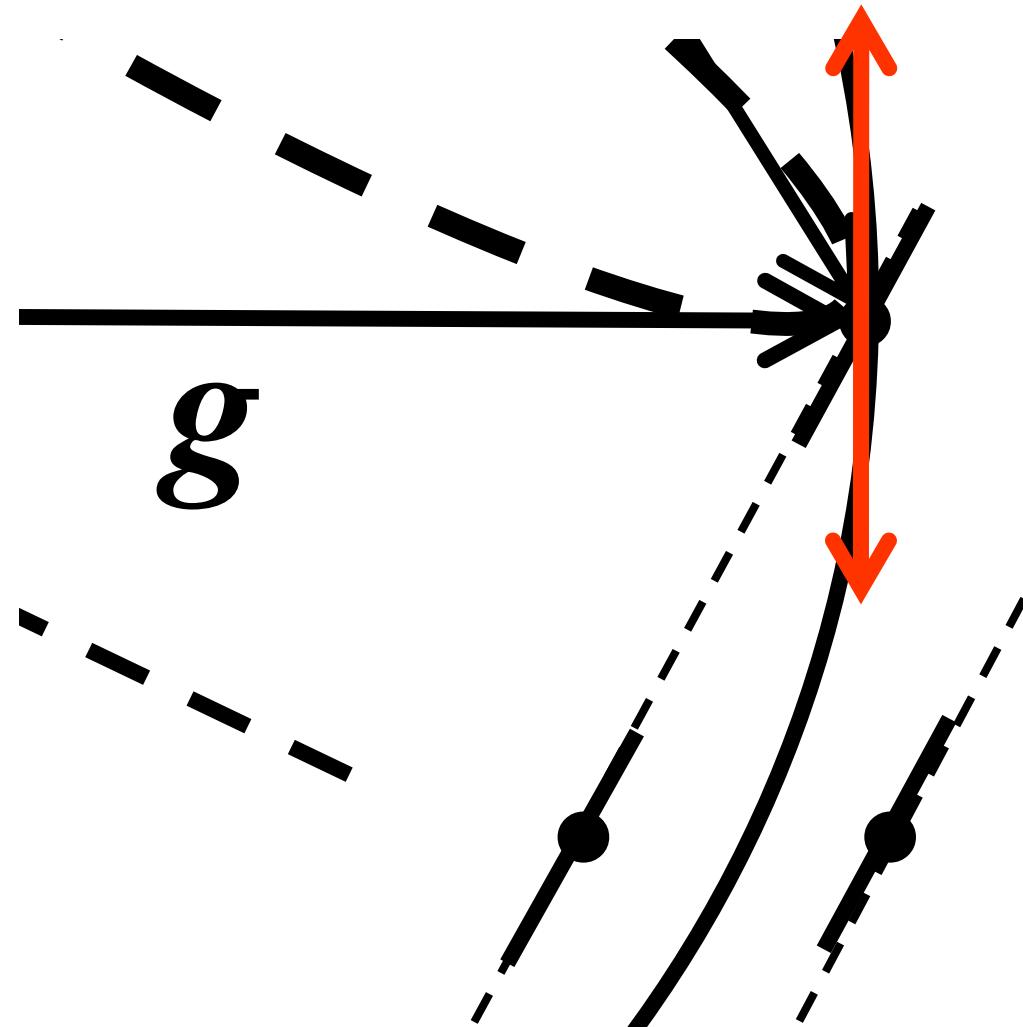
scattered radiation is integrated normal to the **g vector**

$$I(s_g) = \iint I(s_\omega, s_\eta, s_g) ds_\omega ds_\eta$$

Correlation between streaking and line broadening



integration normal to the *g vector*



It can been shown that:

transformation between

L and g is *linear*

to a *very good* approximation

L into *g* transformation for twinning on close-packed planes

$$FWHM_g(\beta) = \frac{1}{g} \left| \frac{h+k+l}{a^2} \right| FWHM_L(\beta)$$

$$FWHM_L = \frac{2\beta}{D_{tri}\sqrt{1-\beta}}$$

L. Velterop, et. al., *J. Appl. Cryst.* (2000). 33, 296-306

E. Estevez-Rams, et. al., *J. Appl. Cryst.* 34, (2001) 730

L. Balogh, G. Ribárik, T Ungár, *J.Appl.Phys.* 100 (2006) 023512

**L into g transformation for *twinning* on
pyramidal planes in hcp crystals**

$$(10\bar{1}1): \quad FWHM(g) = \frac{2}{g} \left| \frac{4c^2h + 2c^2k - 3a^2l}{6a^2c^2} \right| FWHM_{\text{triclinic}}$$

$$(10\bar{1}2): \quad FWHM(g) = \frac{2}{g} \left| \frac{2c^2h + c^2k + 3a^2l}{3a^2c^2} \right| FWHM_{\text{triclinic}}$$

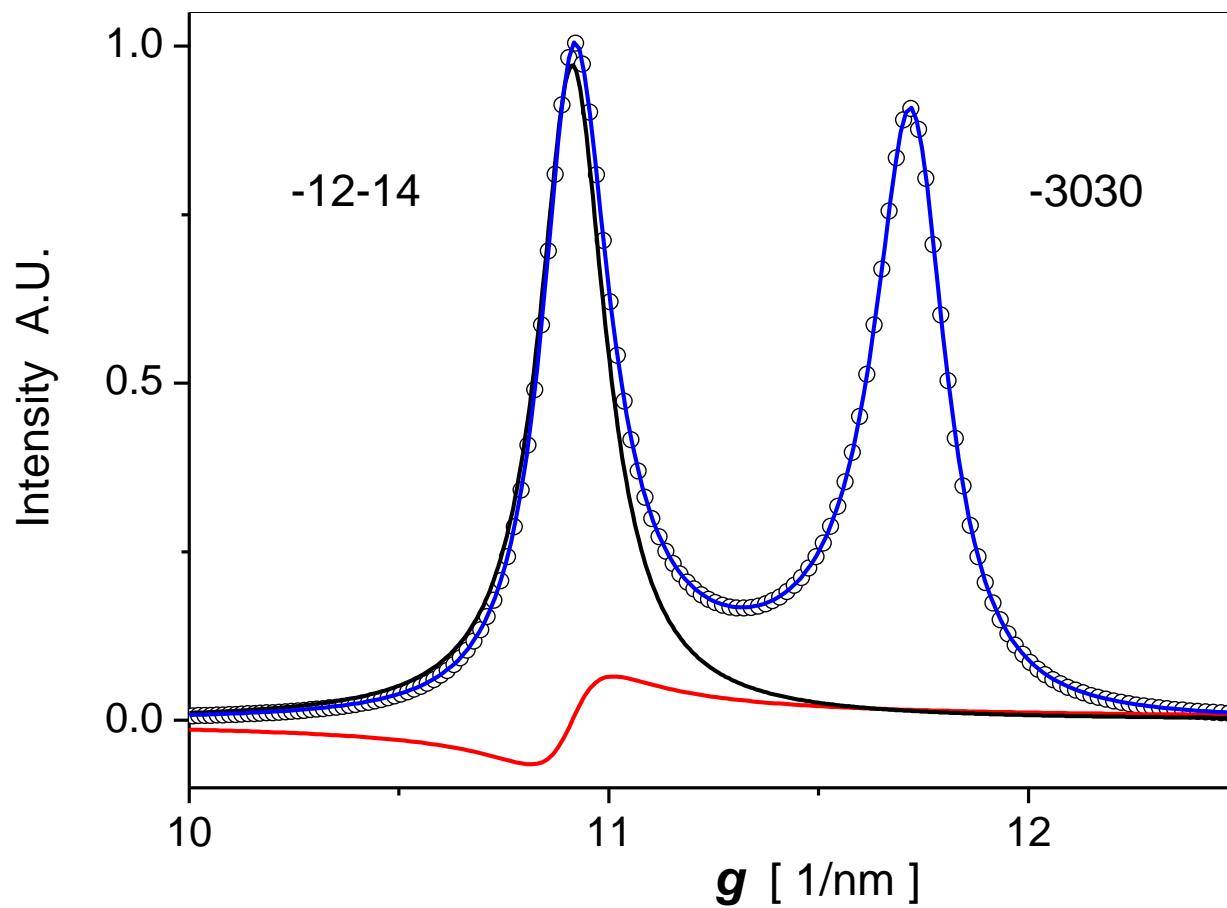
$$(11\bar{2}1): \quad FWHM(g) = \frac{2}{g} \left| \frac{2c^2h + 2c^2k + a^2l}{2a^2c^2} \right| FWHM_{\text{triclinic}}$$

$$(11\bar{2}2): \quad FWHM(g) = \frac{2}{g} \left| \frac{c^2h + c^2k + a^2l}{a^2c^2} \right| FWHM_{\text{triclinic}}$$

$$FWHM_{\text{triclinic}} = \frac{2\beta}{D_{tri}\sqrt{1-\beta}}$$

Ti: *pyramidal* twin plane: $11\bar{2}2$

$\{11.4\}$ reflection: $\bar{1}2\bar{1}4$ sub-reflection (parent crystal)
 $\{31.0\}$ reflection: $\bar{3}030$ sub-reflection (twin crystal)



Profile functions for twinning

along the *streaks* there is *NO hkl dependence*,

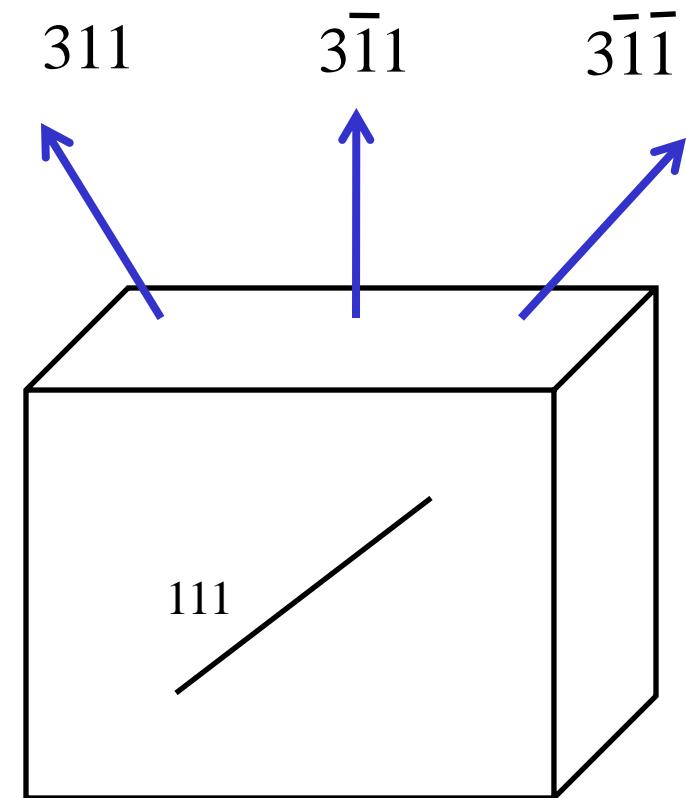
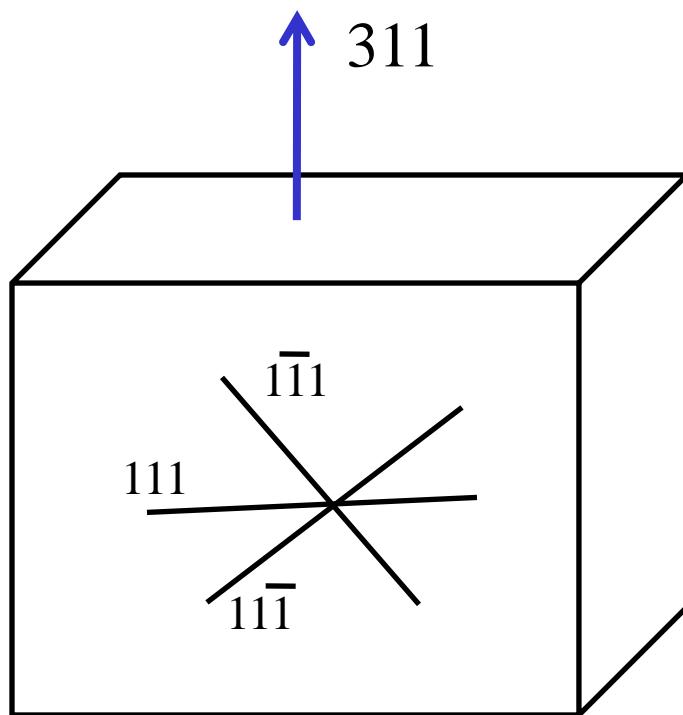
up to the asymmetry

L into *g* transformation

produces the *hkl dependence*

Resultant or measured profiles are
the *sum* of *sub-profiles*

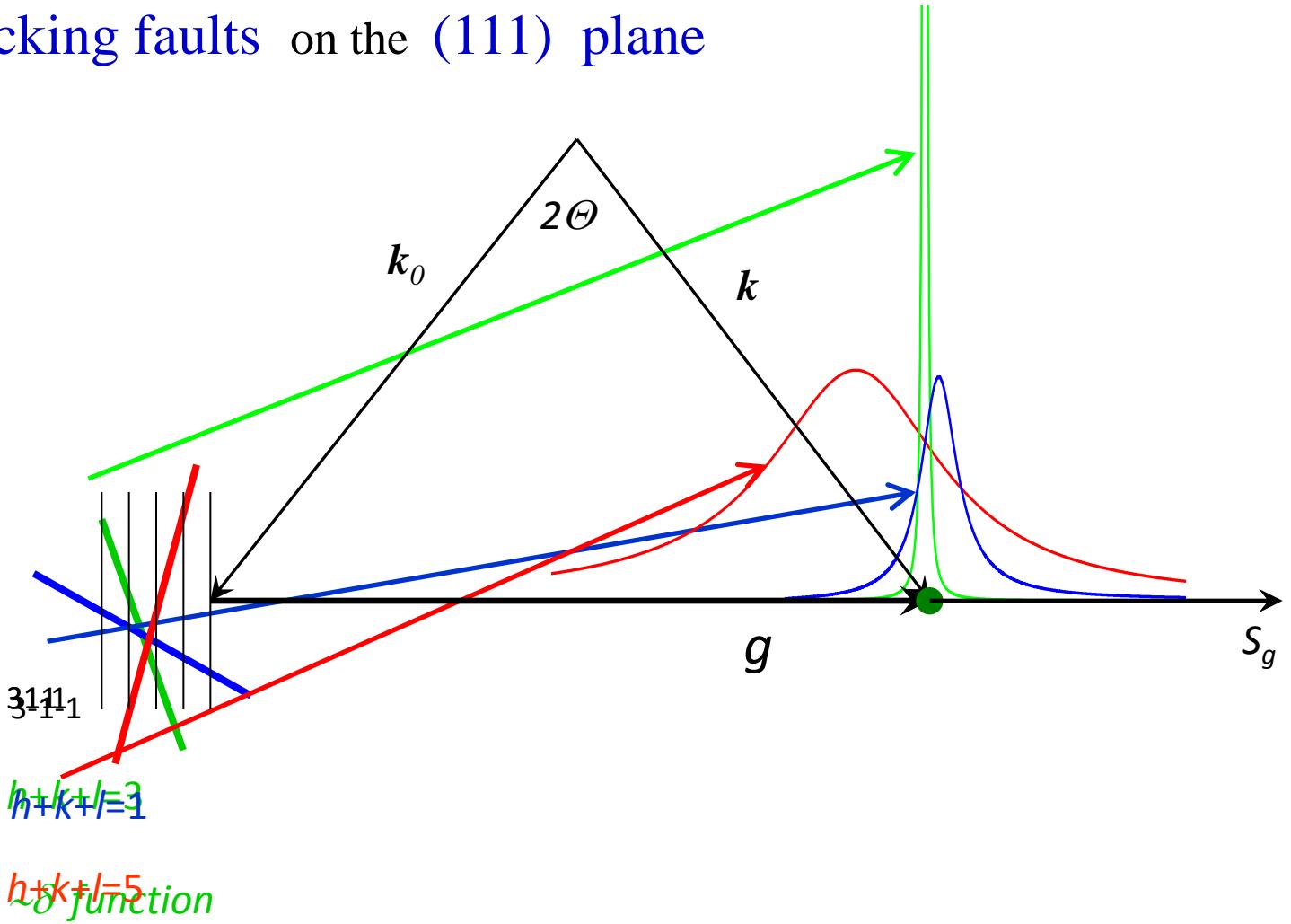
Summation of sub-profiles



either

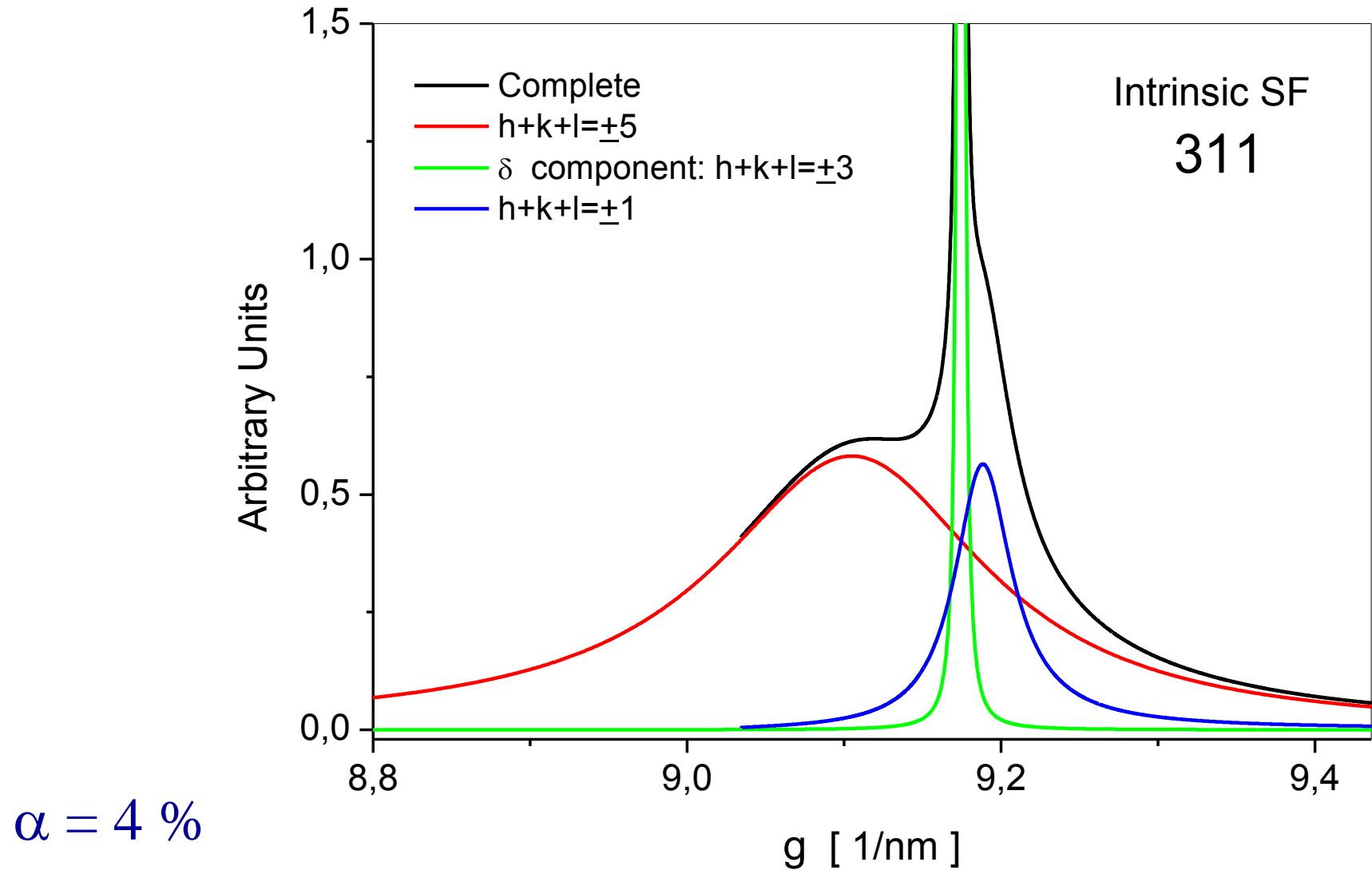
or

intrinsic stacking faults on the (111) plane



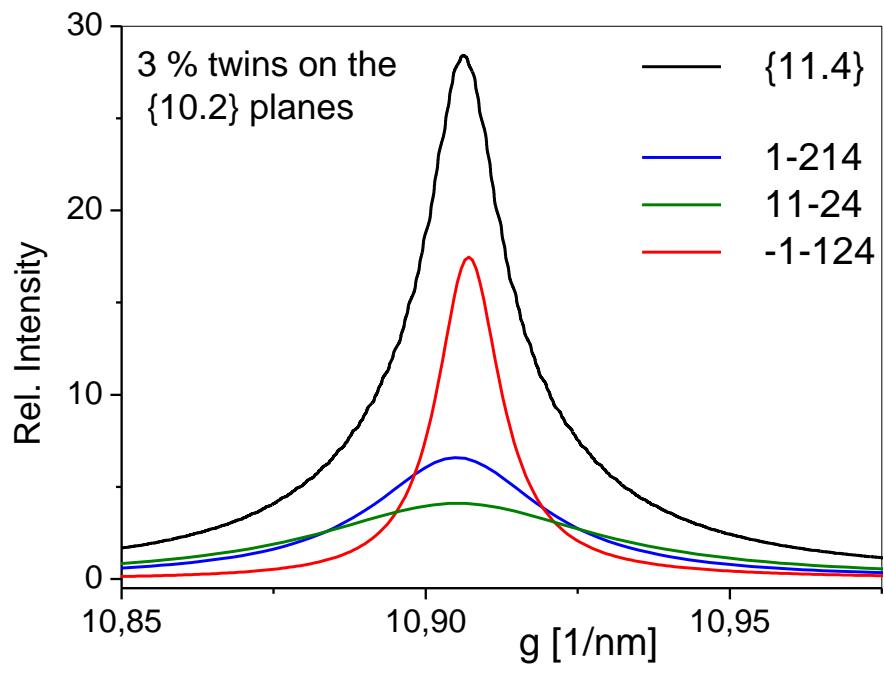
the **311** type sub-reflections in *fcc* crystals

Subreflections according to different hkl permutations

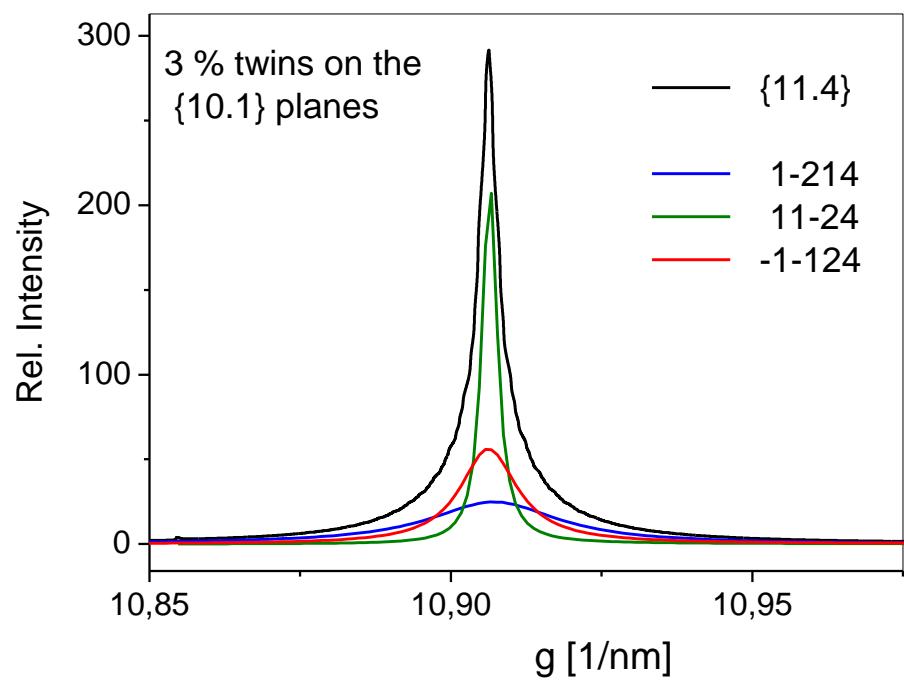


hkl dependence in the $I(g)$ profiles: Ti subreflections and $\{11.4\}$ powder diffraction profiles

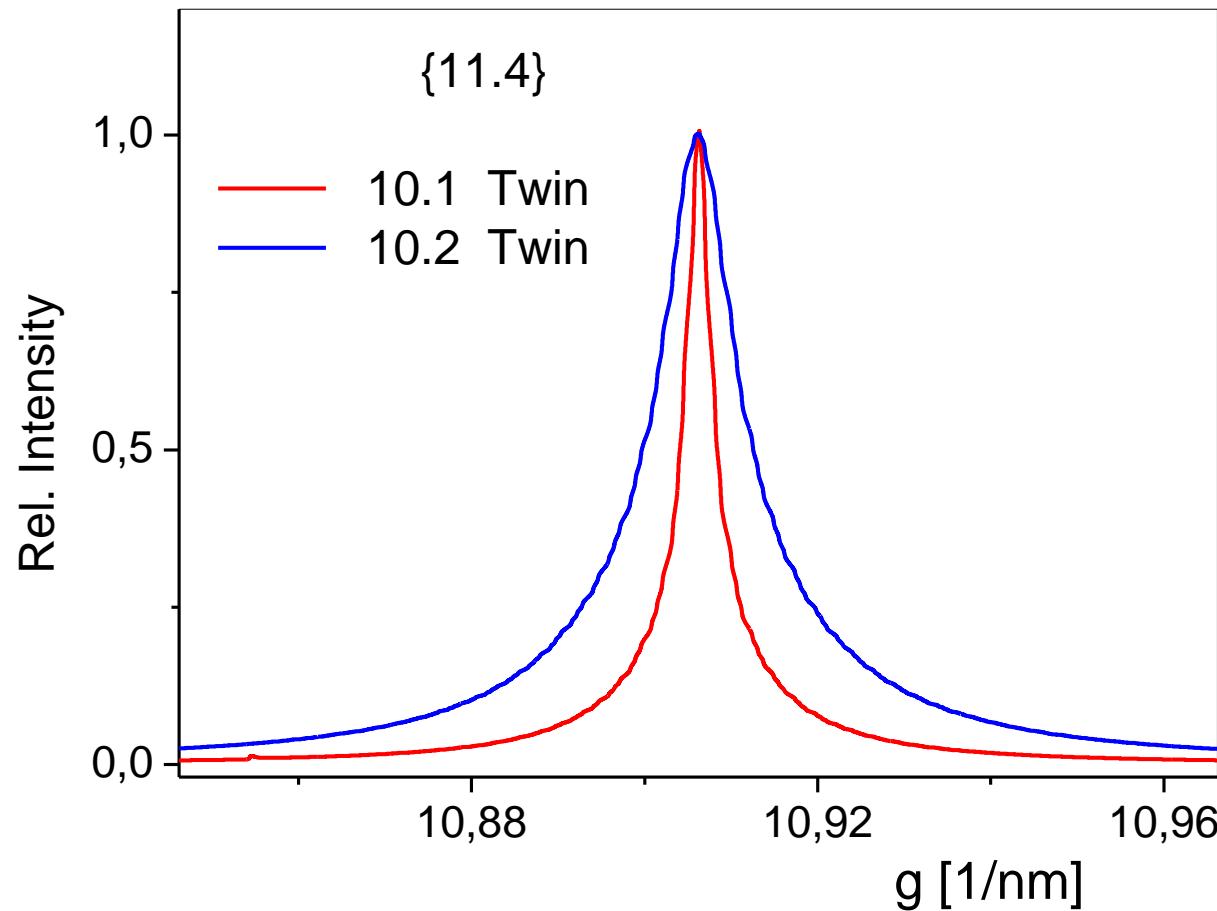
Twin planes: $\{10.2\}$



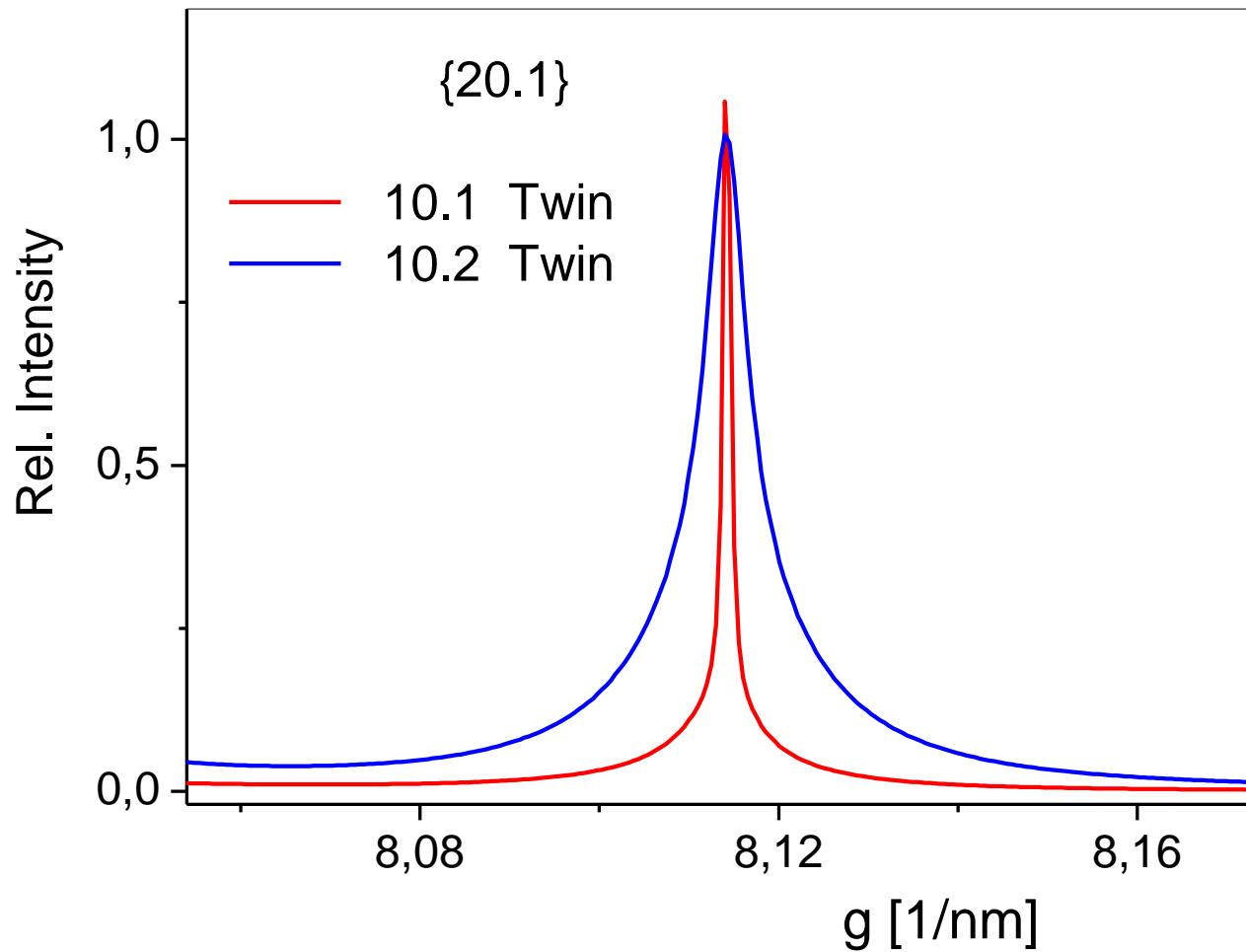
Twin planes: $\{10.1\}$



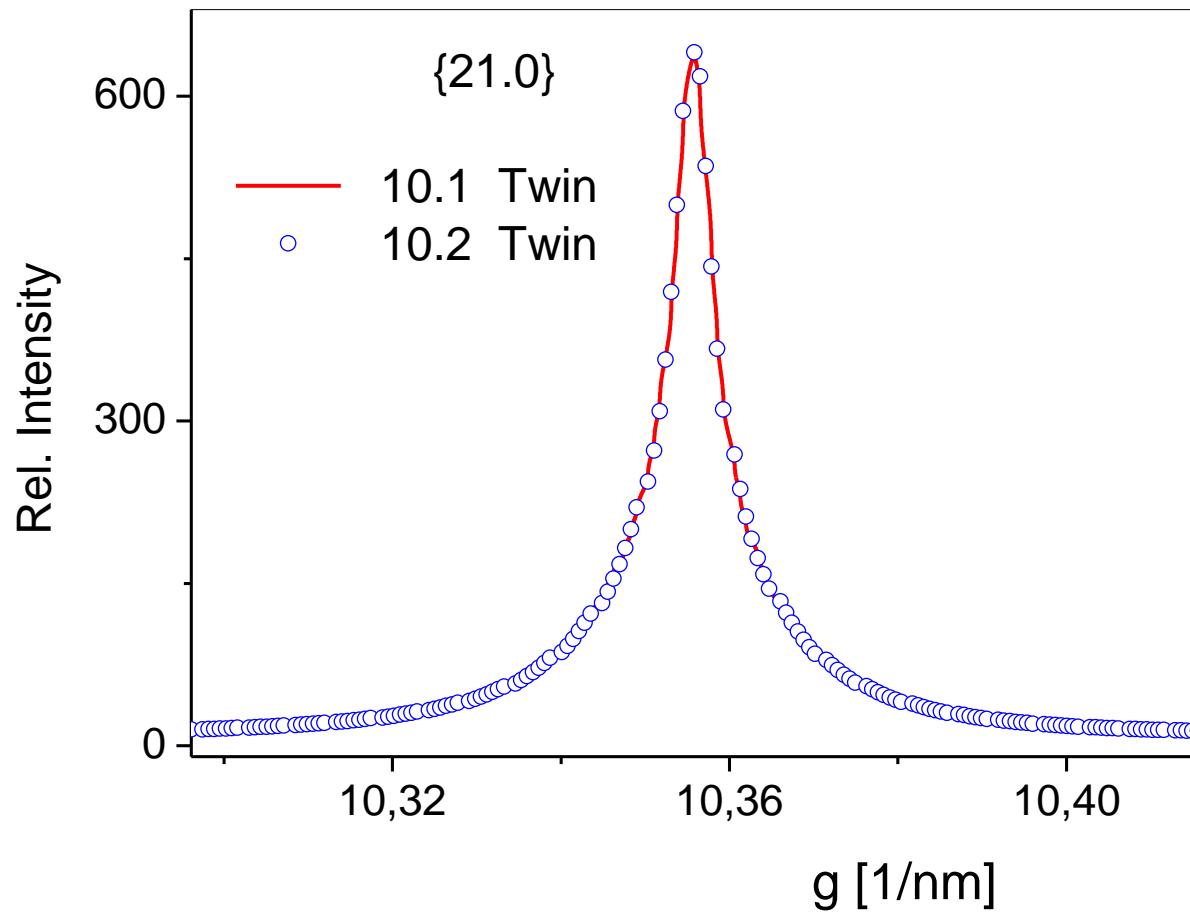
Two different twin planes: {10.1} and {10.2}
{11.4}, {20.1}, {21.0} reflection



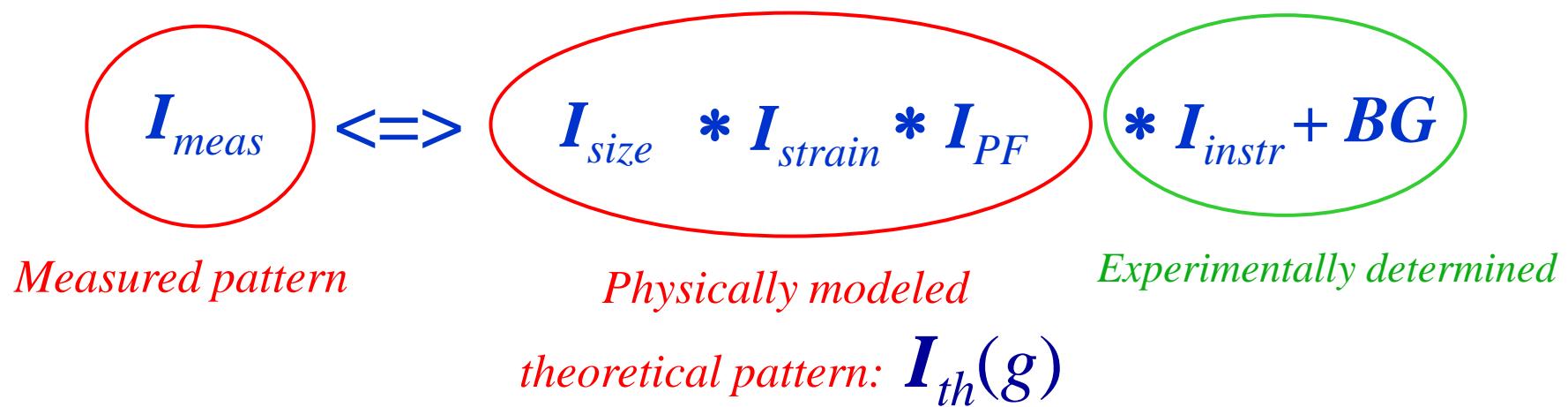
Two different twin planes: {10.1} and {10.2}
{11.4}, {20.1}, {21.0} reflection



Two different twin planes: {10.1} and {10.2}
{11.4}, {20.1}, {21.0} reflection

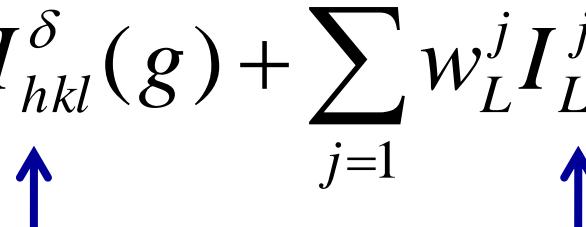


Philosophy of line profile analysis



Profile-functions for planar faults: $I_{hkl}^{PF}(g)$

$$I_{hkl}^{PF}(g) = w_\delta I_{hkl}^\delta(g) + \sum_{j=1}^n w_L^j I_{L,hkl}^j(g)$$



 δ functions sub-profiles

w : fractions, (\sim permutations of hkl)

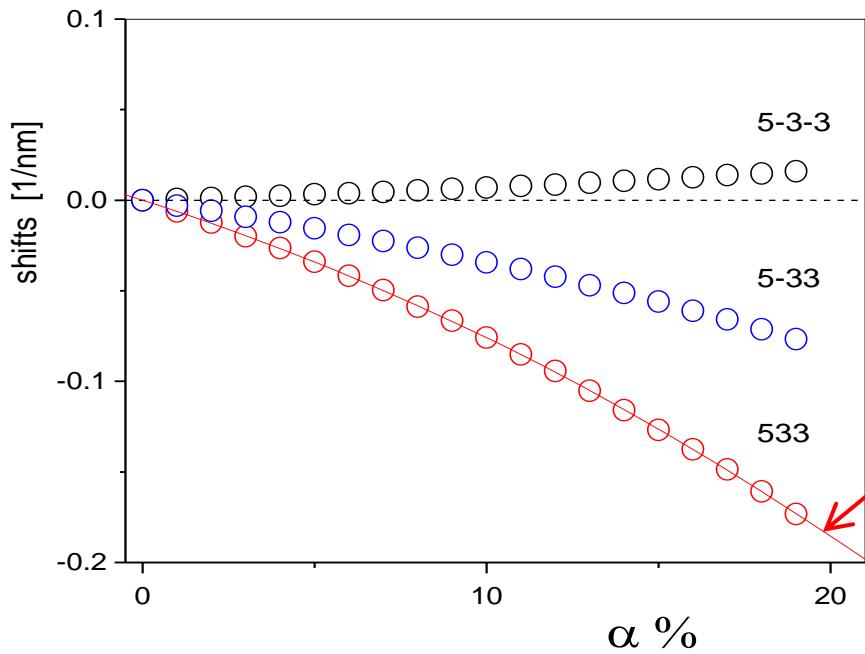
Sub-profiles:

- symmetrical + antisymmetrical

Lorentzian functions

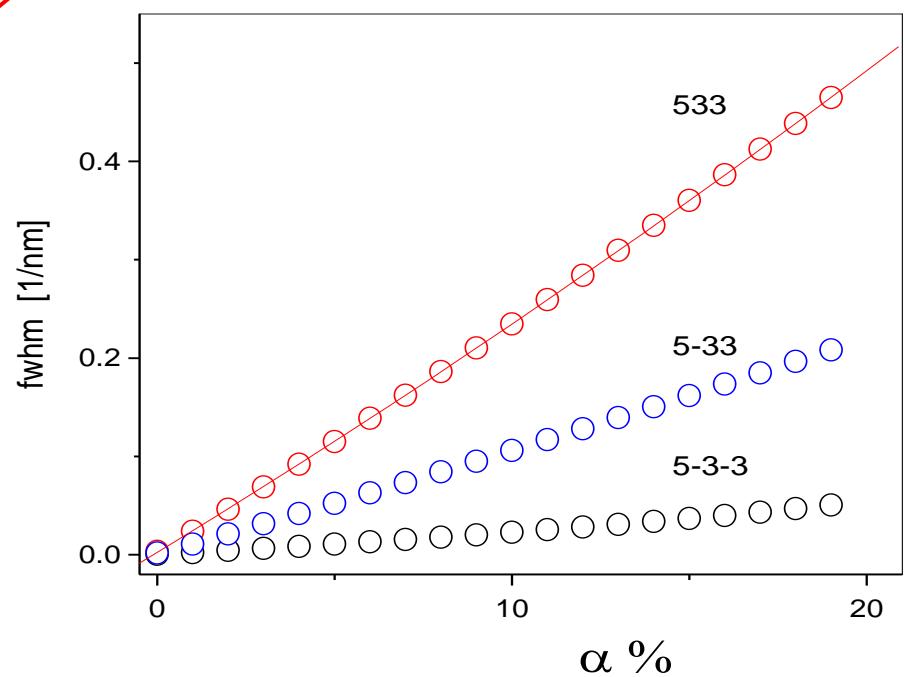
- easy to parameterize

Sub-reflections of the {533} reflection for intrinsic SF



Shifts

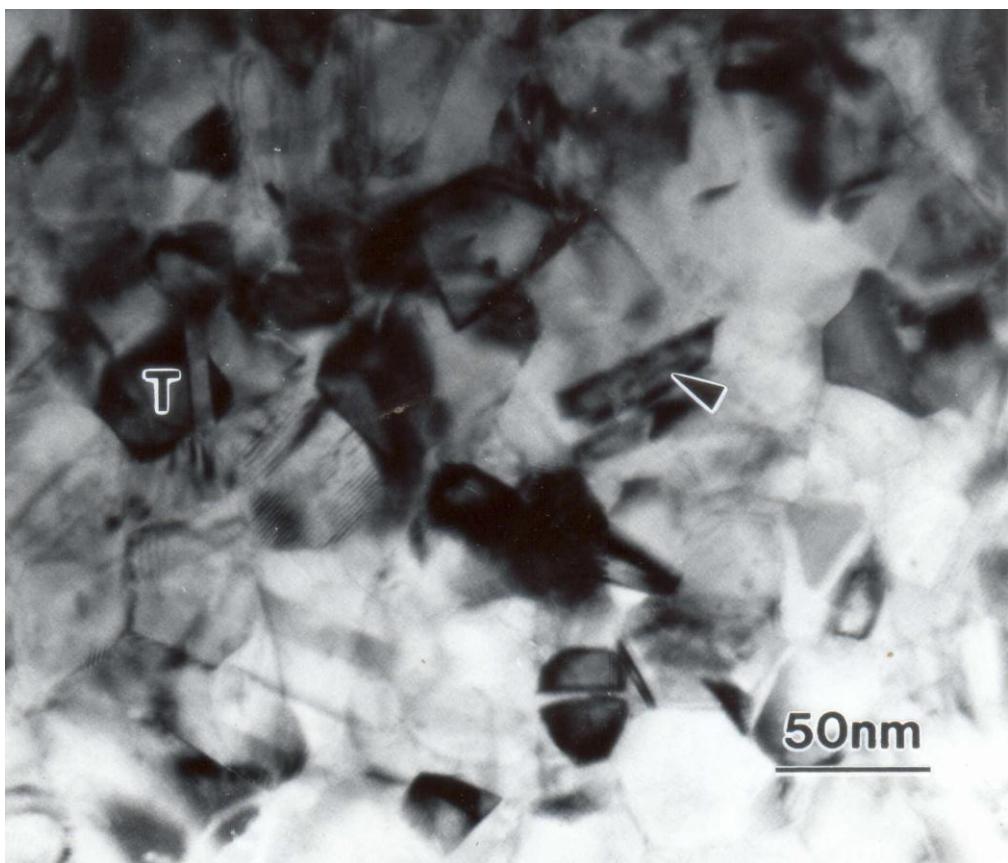
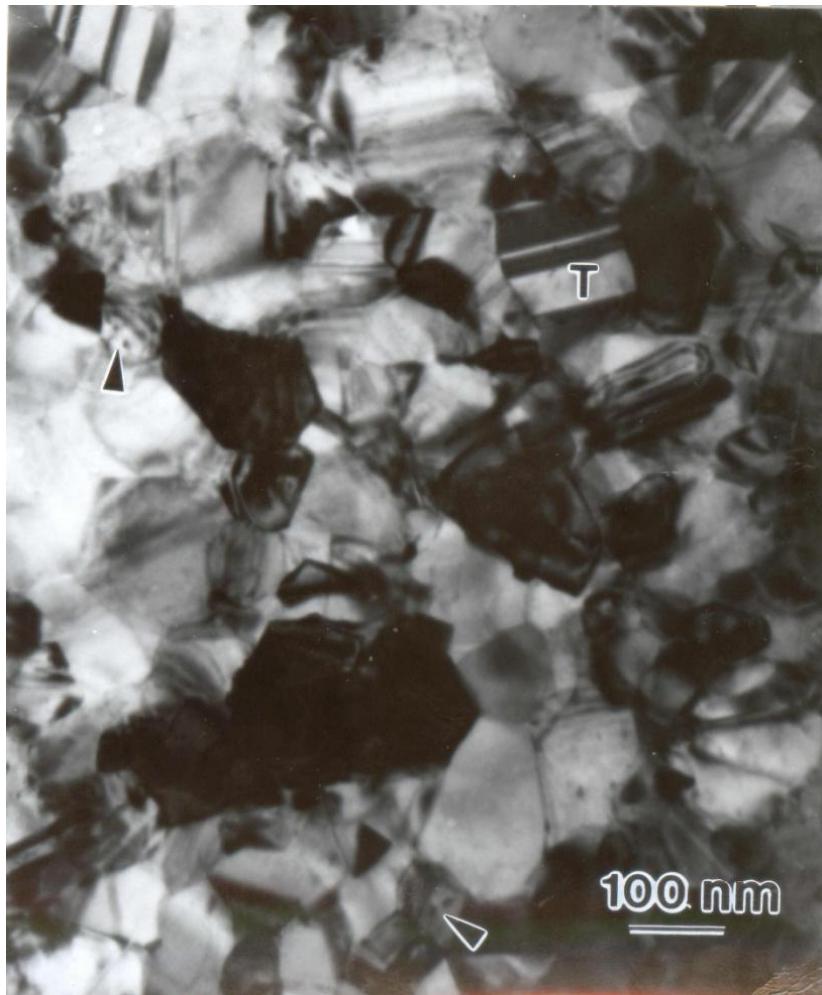
5th order polinomials



Breadths

Inert-Gas condensed nanocrystalline copper

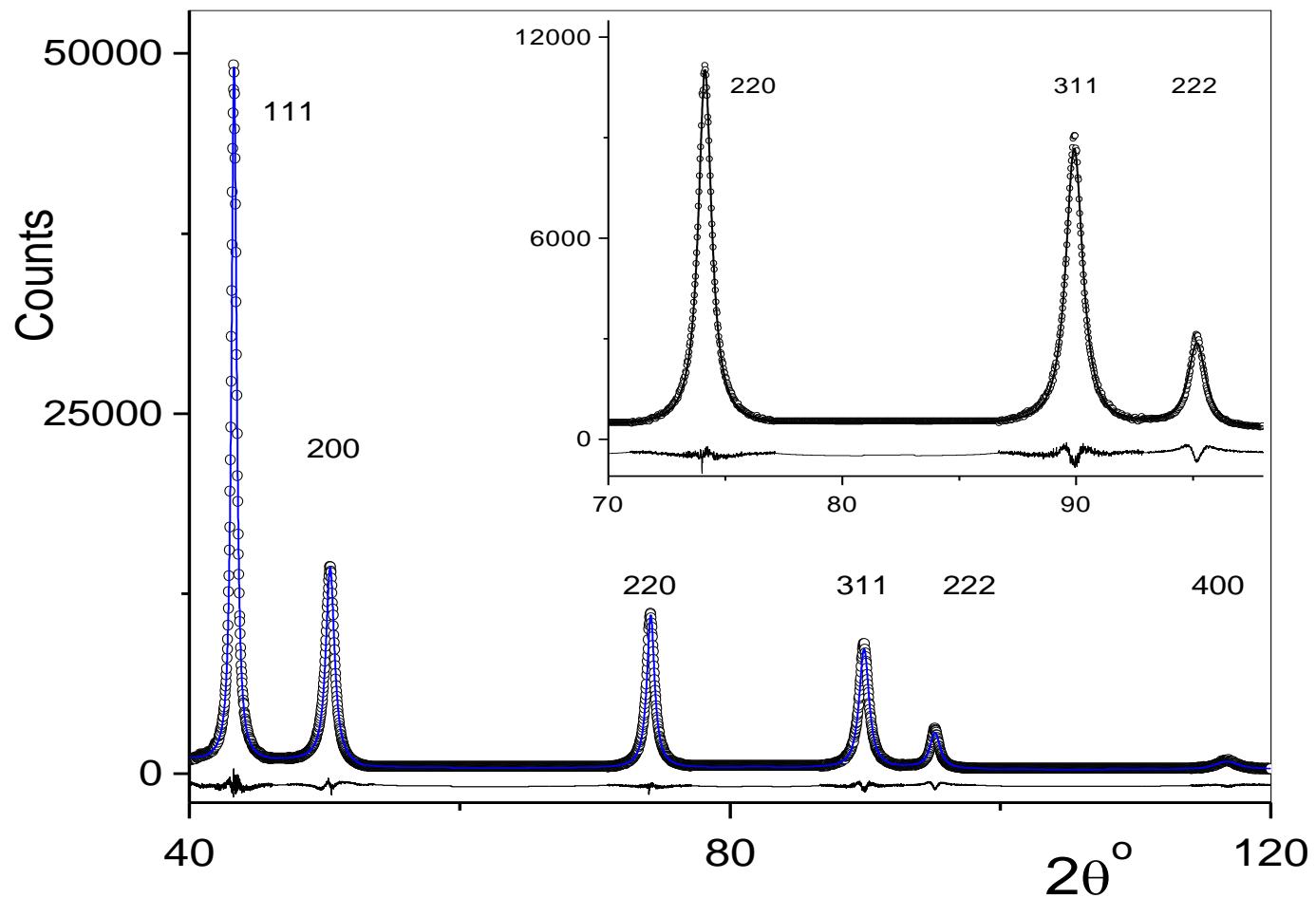
G. Sanders, G. E. Fougere, L. J. Thompson, J. A. Eastman, J. R. Weertman, *Nanostruct. Mater.* 8, (1997) 243.



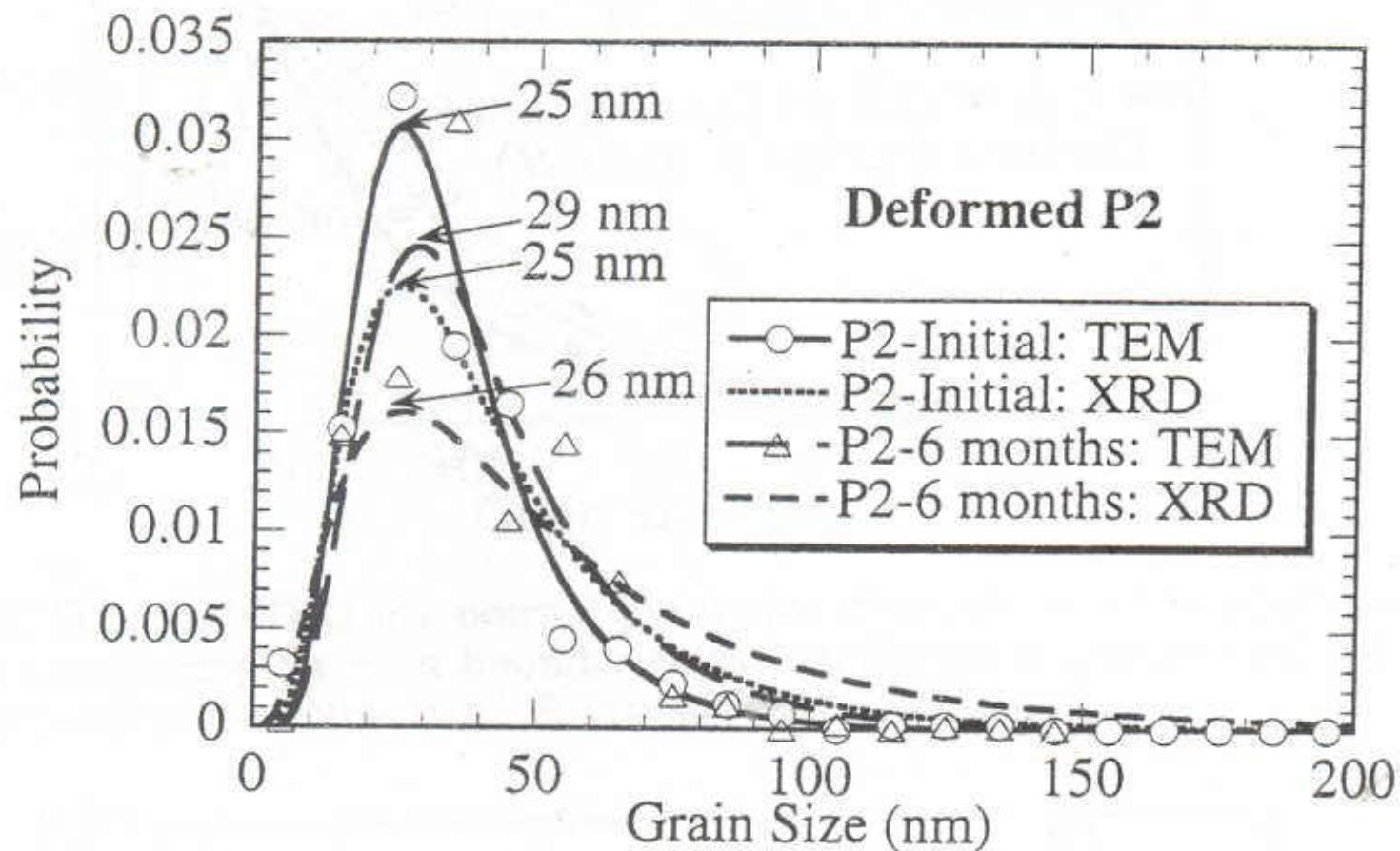
Inert-Gas condensed nanocrystalline copper

T.Ungár, S.Ott, P.G.Sanders, A.Borbély, J.R.Weertman, *Acta Materialia*, **10**, 3693-3699 (1998)

L. Balogh, G. Ribárik, T Ungár, *J.Appl.Phys.* **100** (2006) 023512

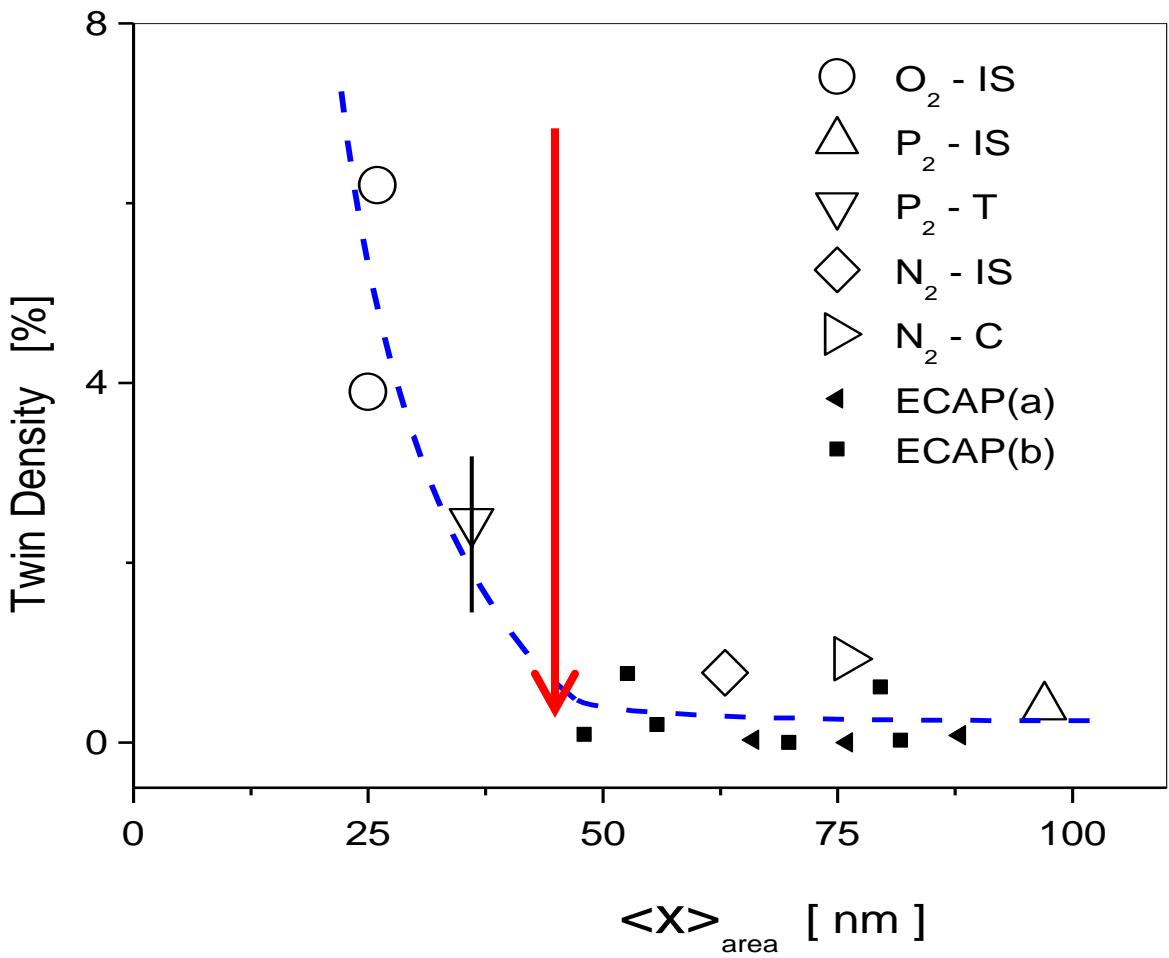


TEM and X-ray grain size



Twin density vs. crystallite size: in Cu

L. Balogh, G. Ribárik, T. Ungár, *J.Appl.Phys.* 100 (2006) 023512



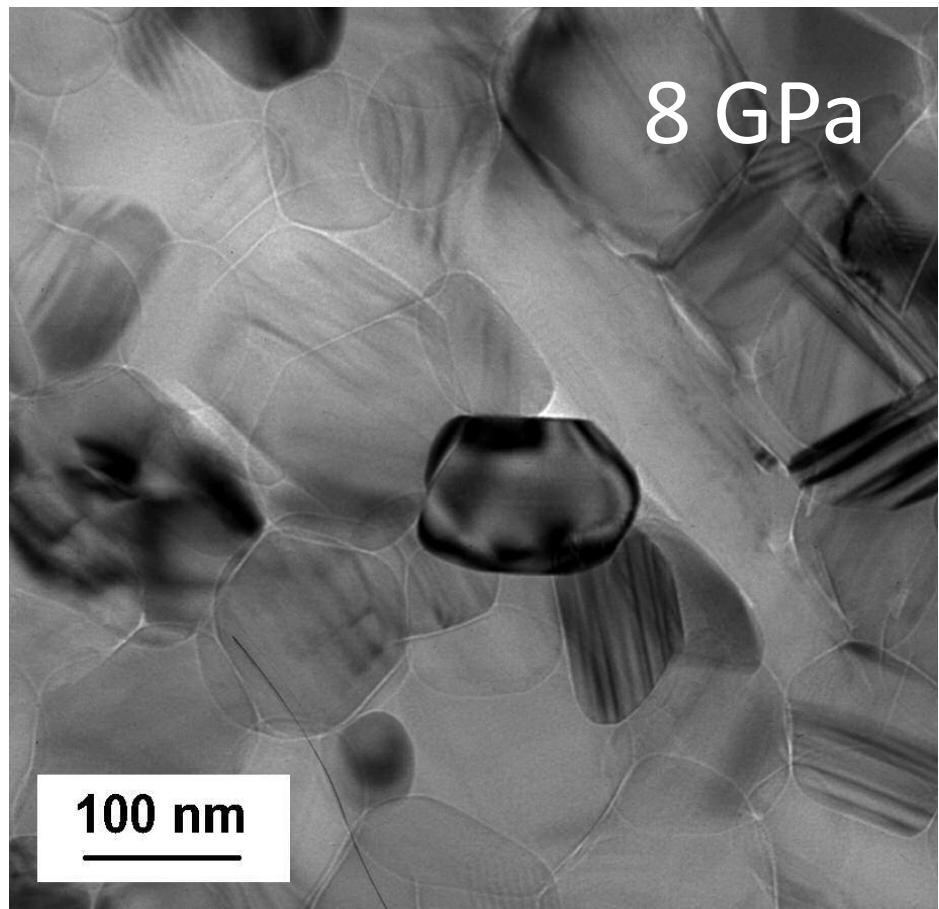
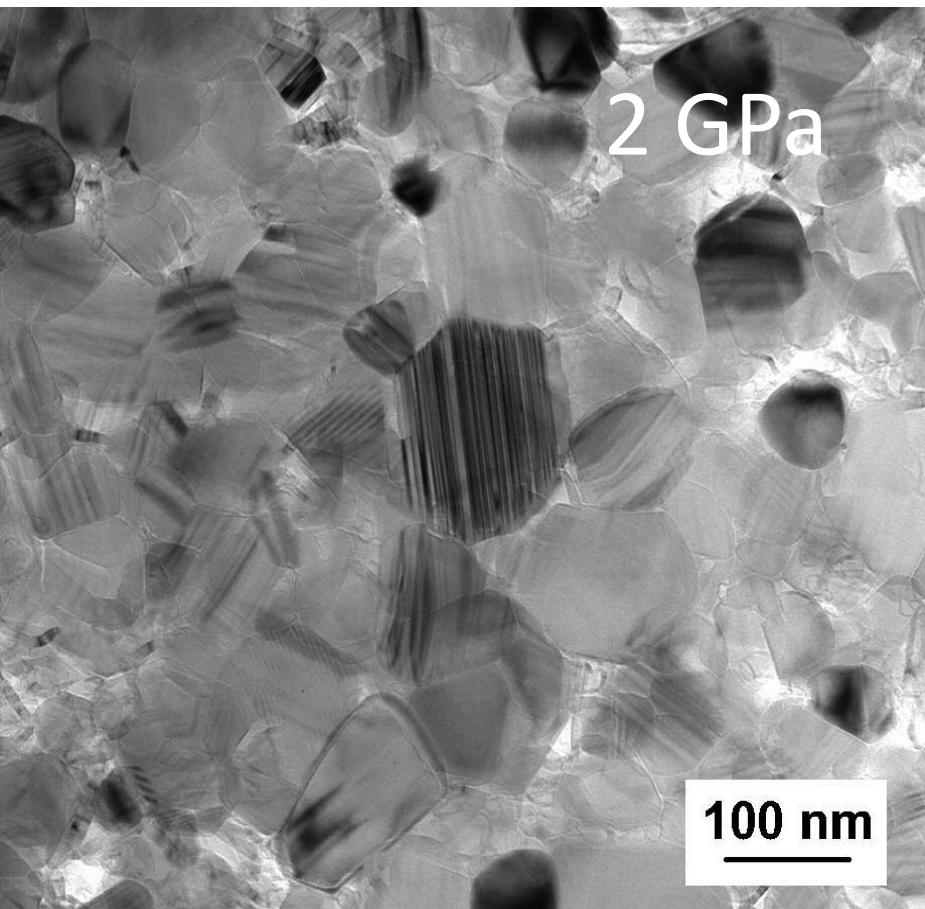
Twinning in Cu:

below ~ 40 nm

Sintering nanocrystalline SiC

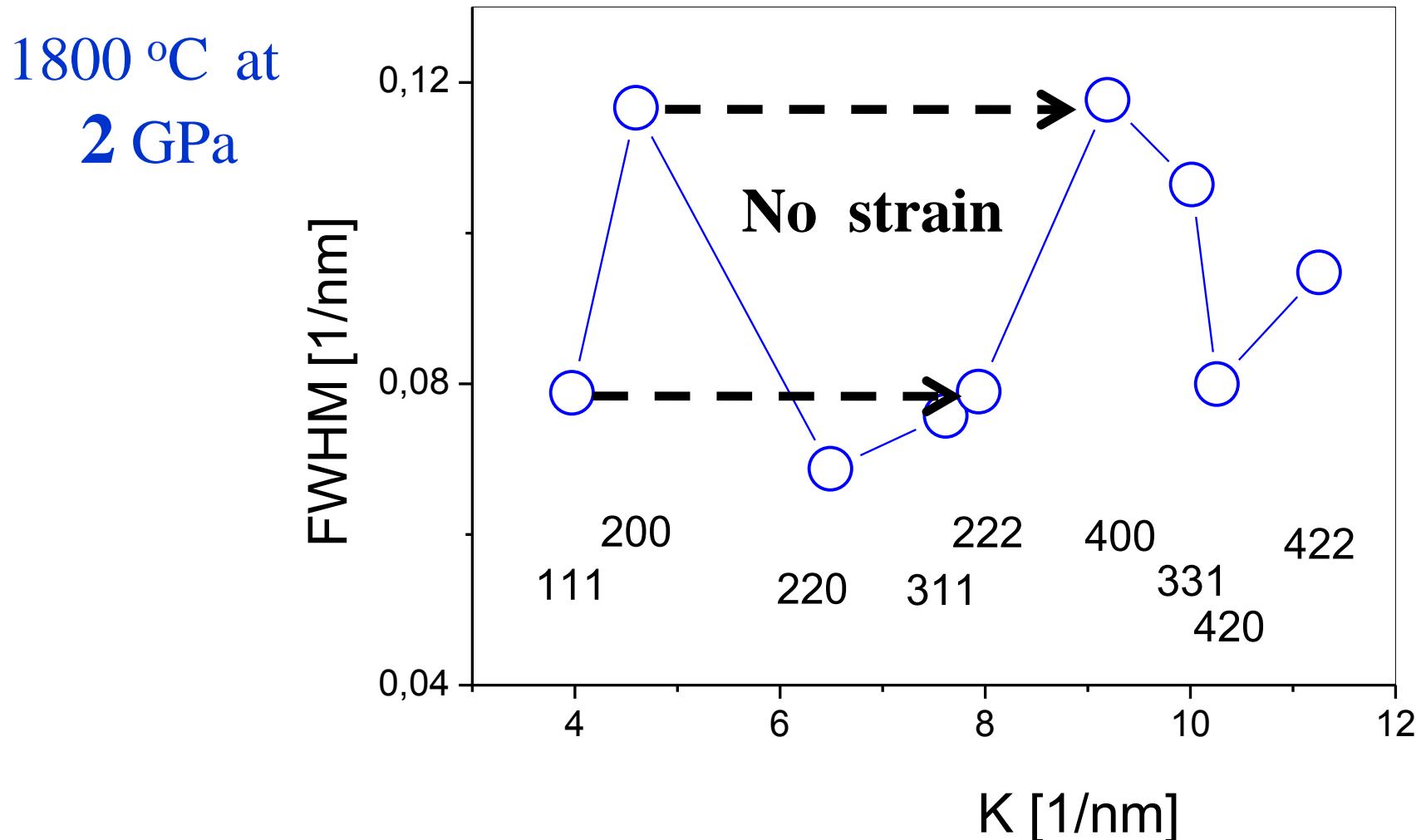
J. Gubicza, S. Nauyoks, L. Balogh, J. Lábár, T. W. Zerda, T. Ungár, *J. Mater. Res.* **22** (2007) 1314

T_{sinter} 1800 °C



Sintering nanocrystalline SiC

J. Gubicza, S. Nauyoks, L. Balogh, J. Lábár, T. W. Zerda, T. Ungár, *J. Mater. Res.* **22** (2007) 1314

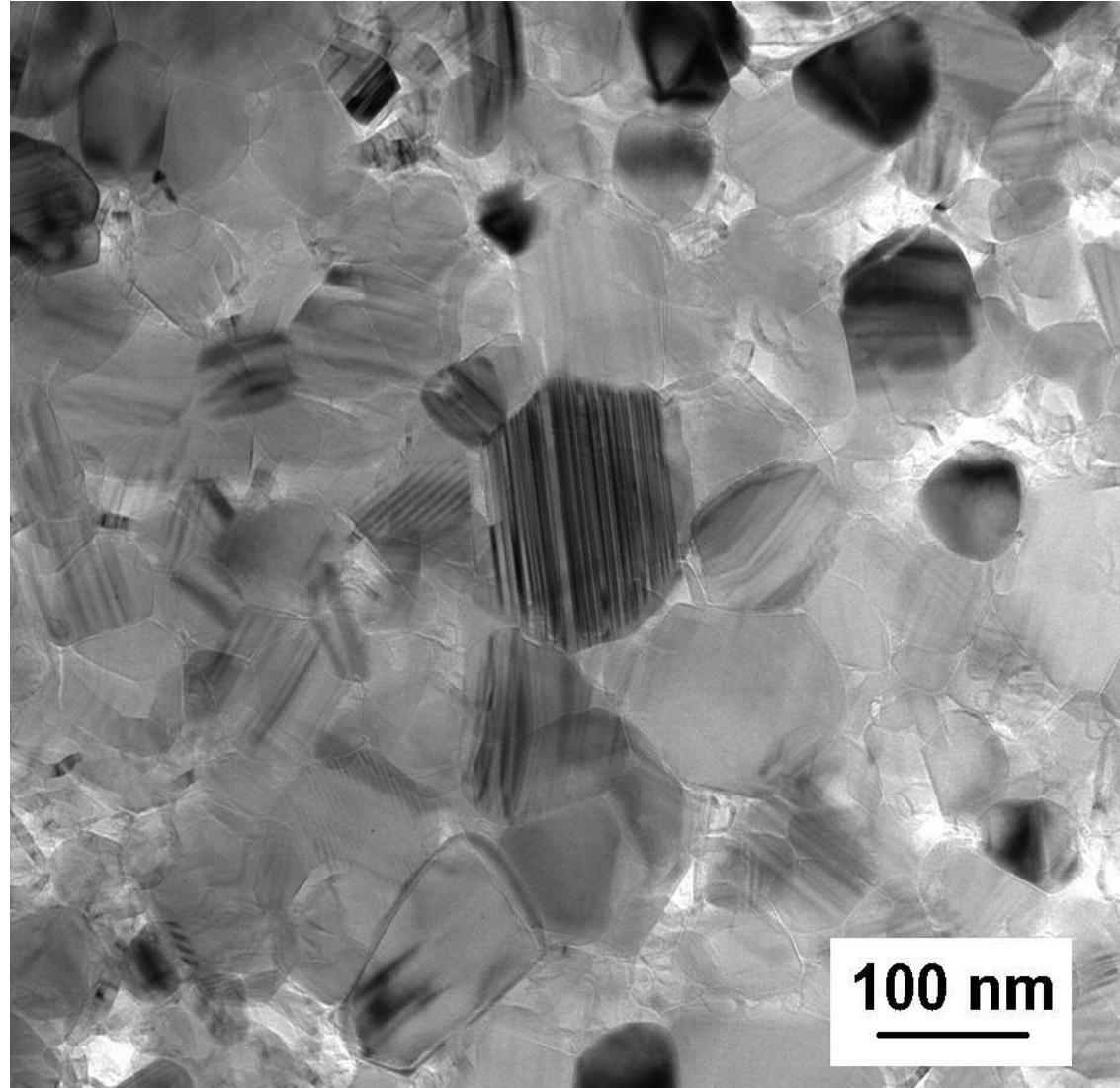


Sintering nanocrystalline SiC

J. Gubicza, S. Nauyoks, L. Balogh, J. Lábár, T. W. Zerda, T. Ungár, *J. Mater. Res.* **22** (2007) 1314

1800 °C at
2 GPa

Equiaxed grains
with
twin boundaries

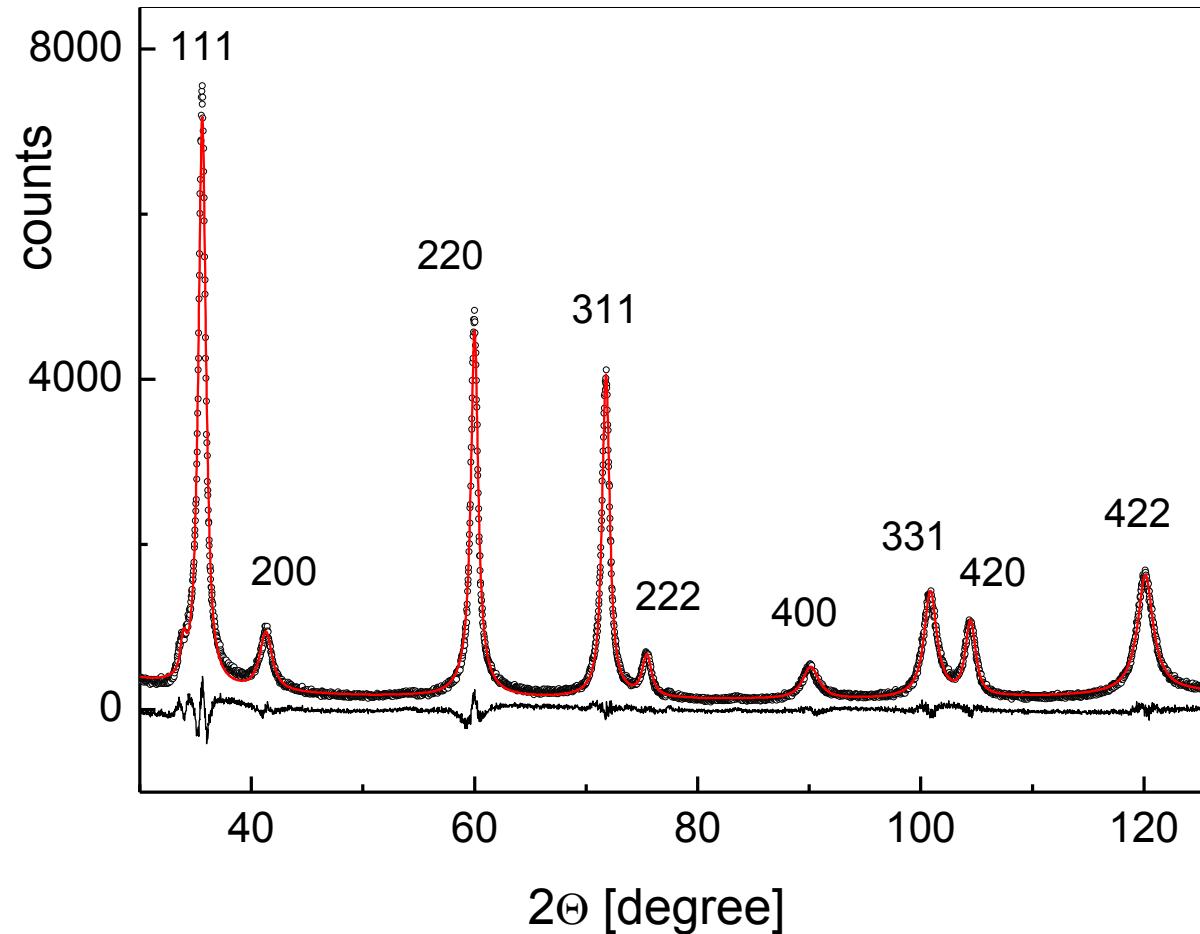


Sintering nanocrystalline SiC

J. Gubicza, S. Nauyoks, L. Balogh, J. Lábár, T. W. Zerda, T. Ungár, *J. Mater. Res.* **22** (2007) 1314

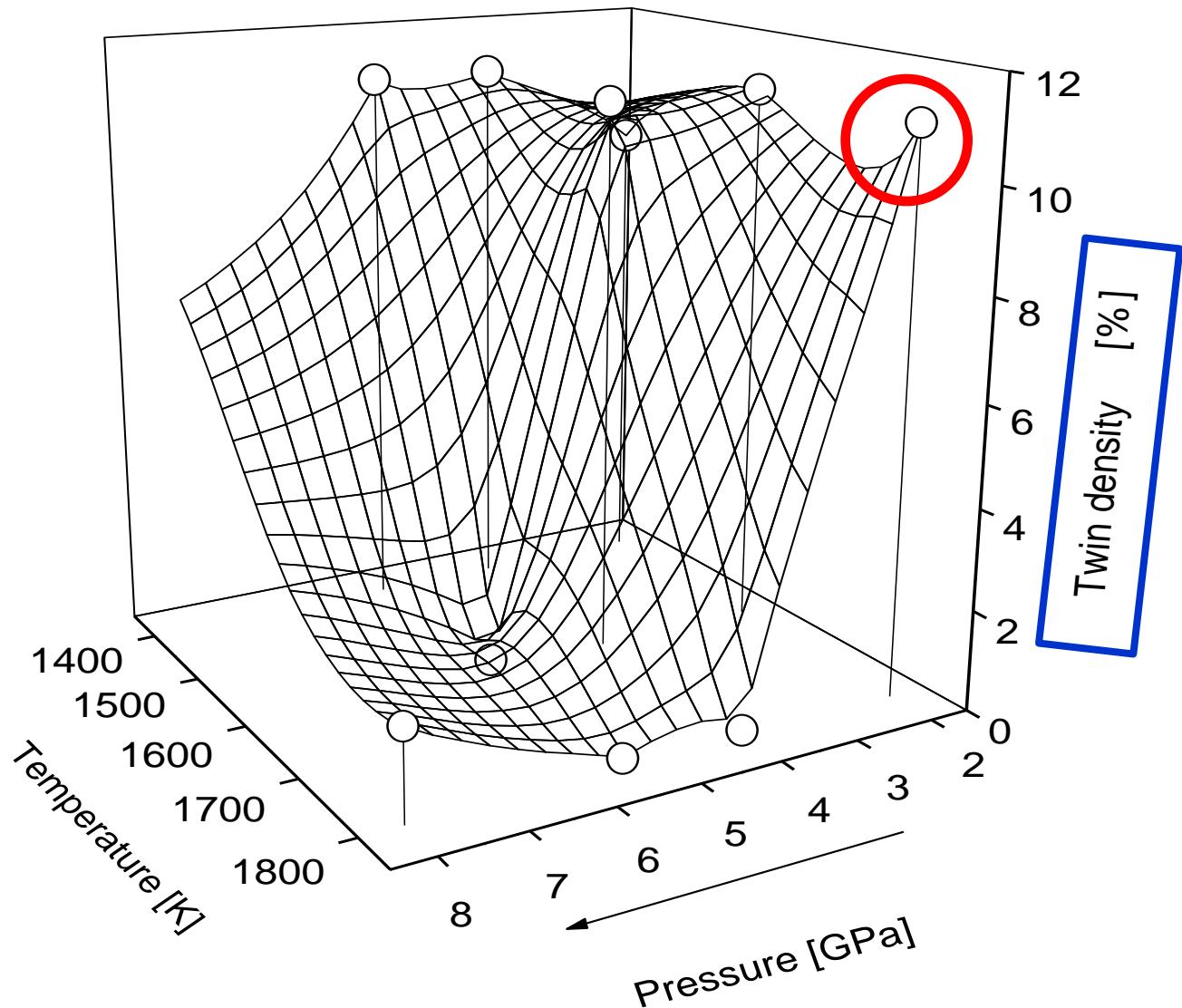
1800 °C at
2 GPa

eCMWP
procedure



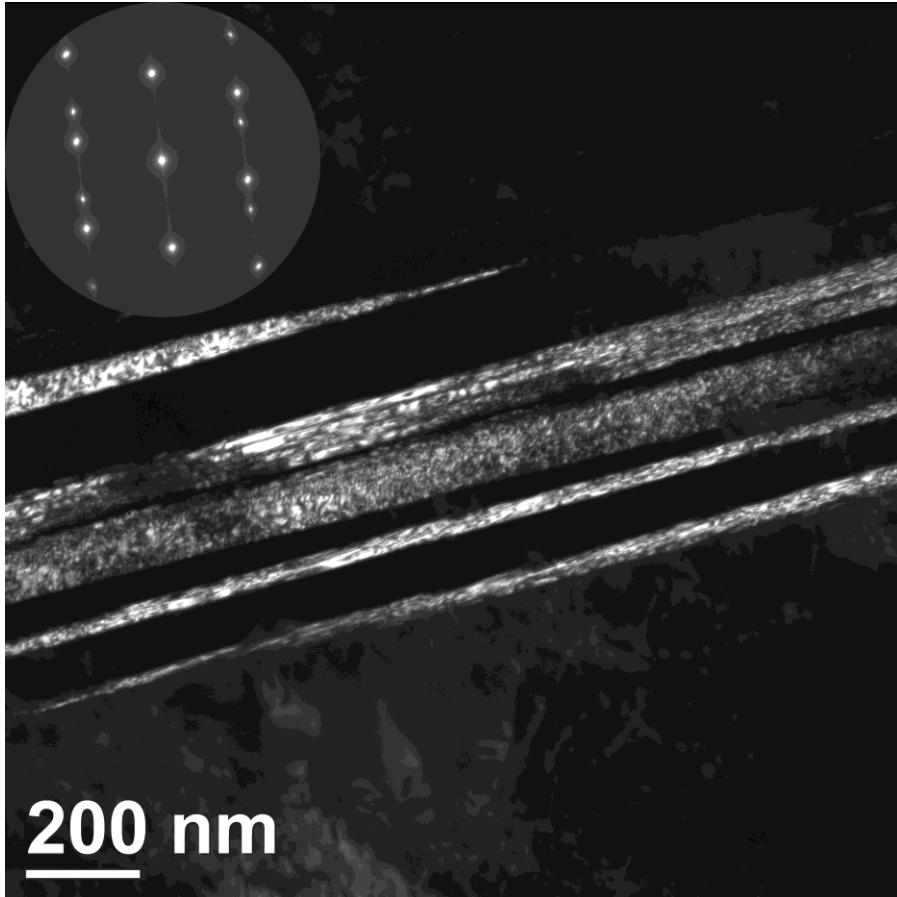
Sintering nanocrystalline SiC

J. Gubicza, S. Nauyoks, L. Balogh, J. Lábár, T. W. Zerda, T. Ungár, *J. Mater. Res.* **22** (2007) 1314

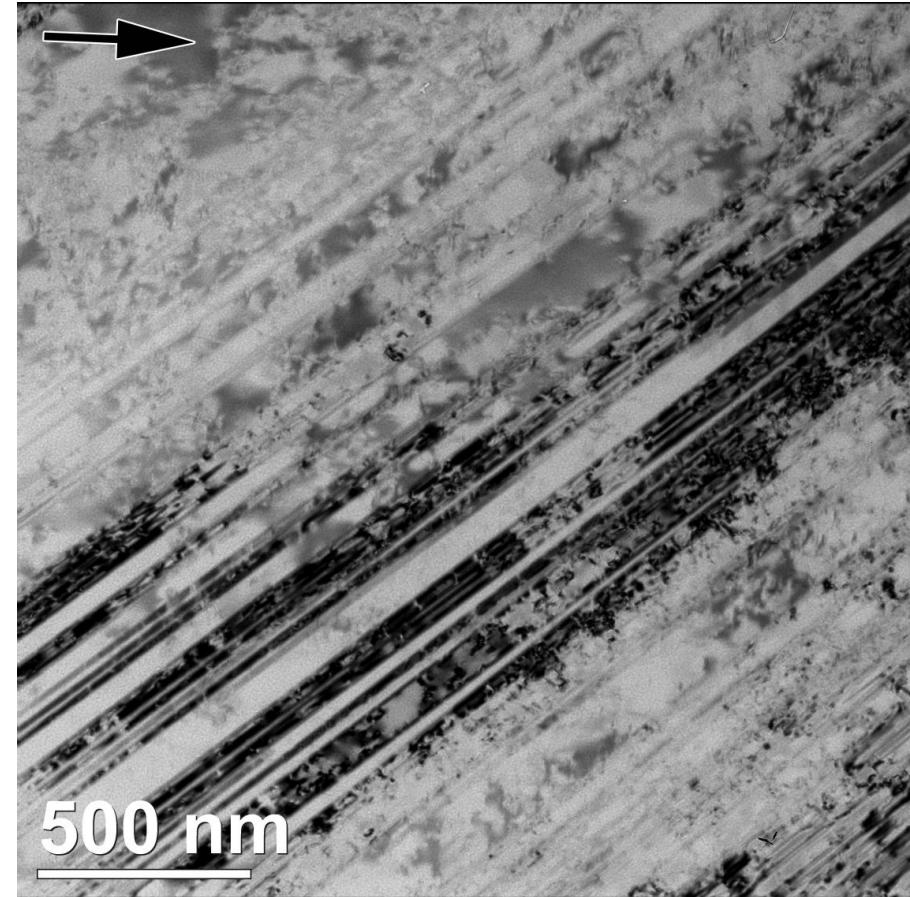


Twinning in tensile deformed TWIP steel

L. Balogh, D.W. Brown, T. Holden, in preparation



$\varepsilon_{\text{eng}} = 33\%$



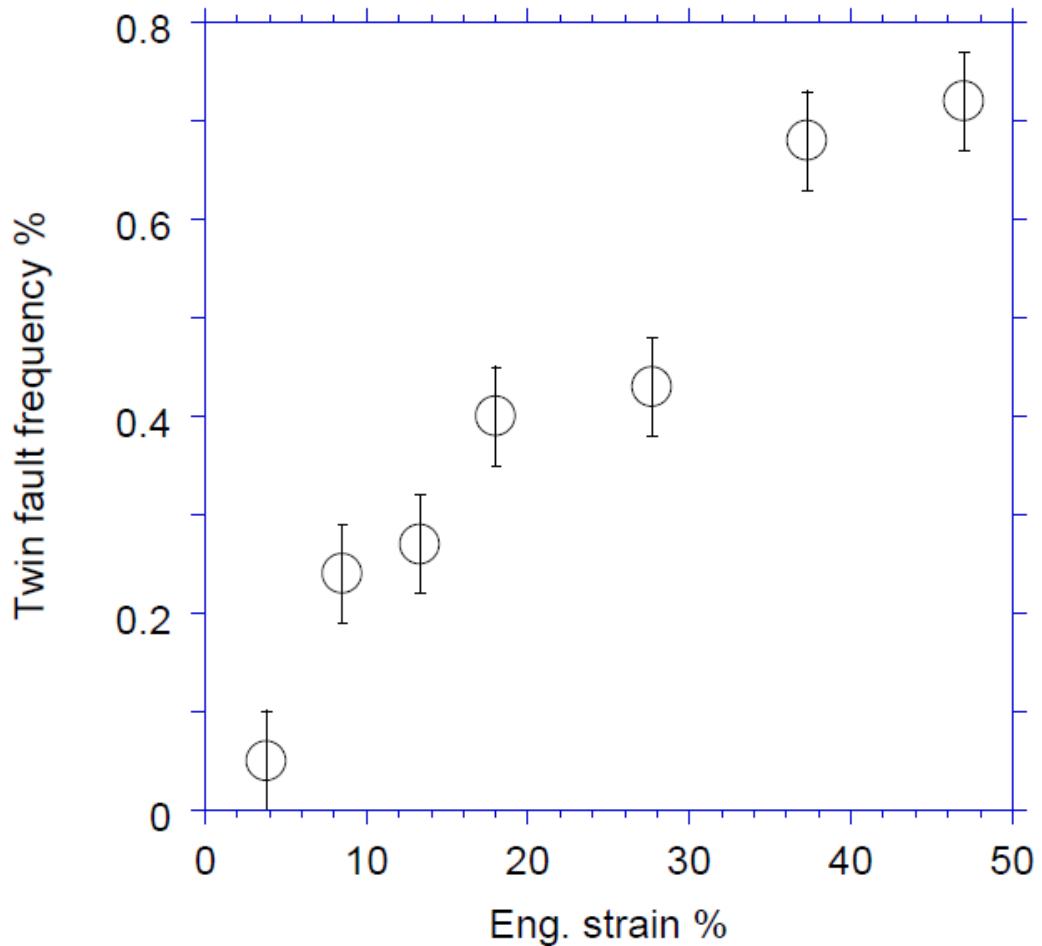
$\varepsilon_{\text{eng}} = 42\%$

Twinning in tensile deformed TWIP steel

L. Balogh, D.W. Brown, T. Holden, in preparation

eCMWP
procedure

TOF neutron diffraction
at SMARTS
Lujan Neutron Scatt. Cent.
Los Alamos Nat. Lab.



Access to the software package

- CMWP
- eCMWP
- ANIZC
- planar-defect parameter-files

<http://www.renyi.hu/cmwp>

<http://metal.elte.hu/anizc/>

<http://metal.elte.hu/~levente/stacking>

Thanks to my coworkers:

Levente Balogh

Gábor Ribárik

Géza Tichy

Thank you

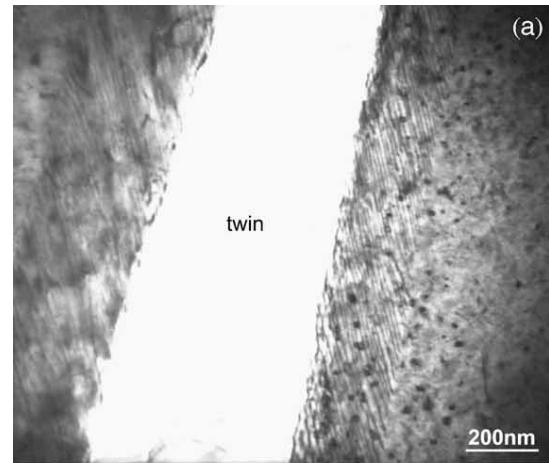
for your

attention

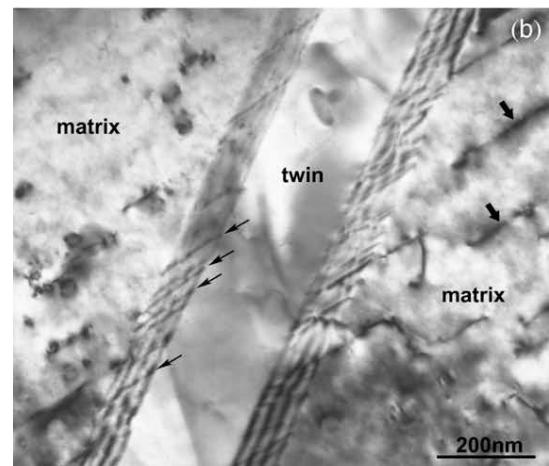
(a)

Dislocations in the pyramidal twin boundaries in Mg

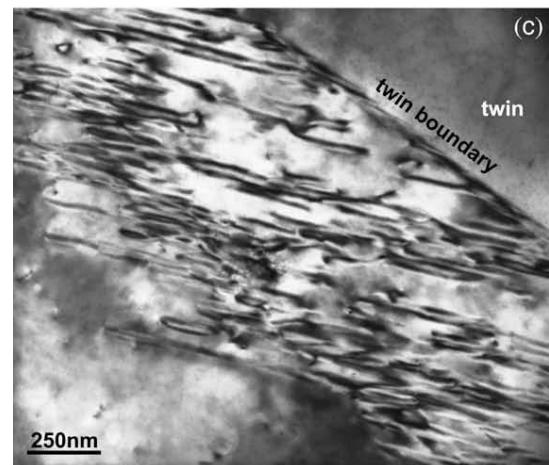
B. Li, E. Ma, AM 2004



(b)

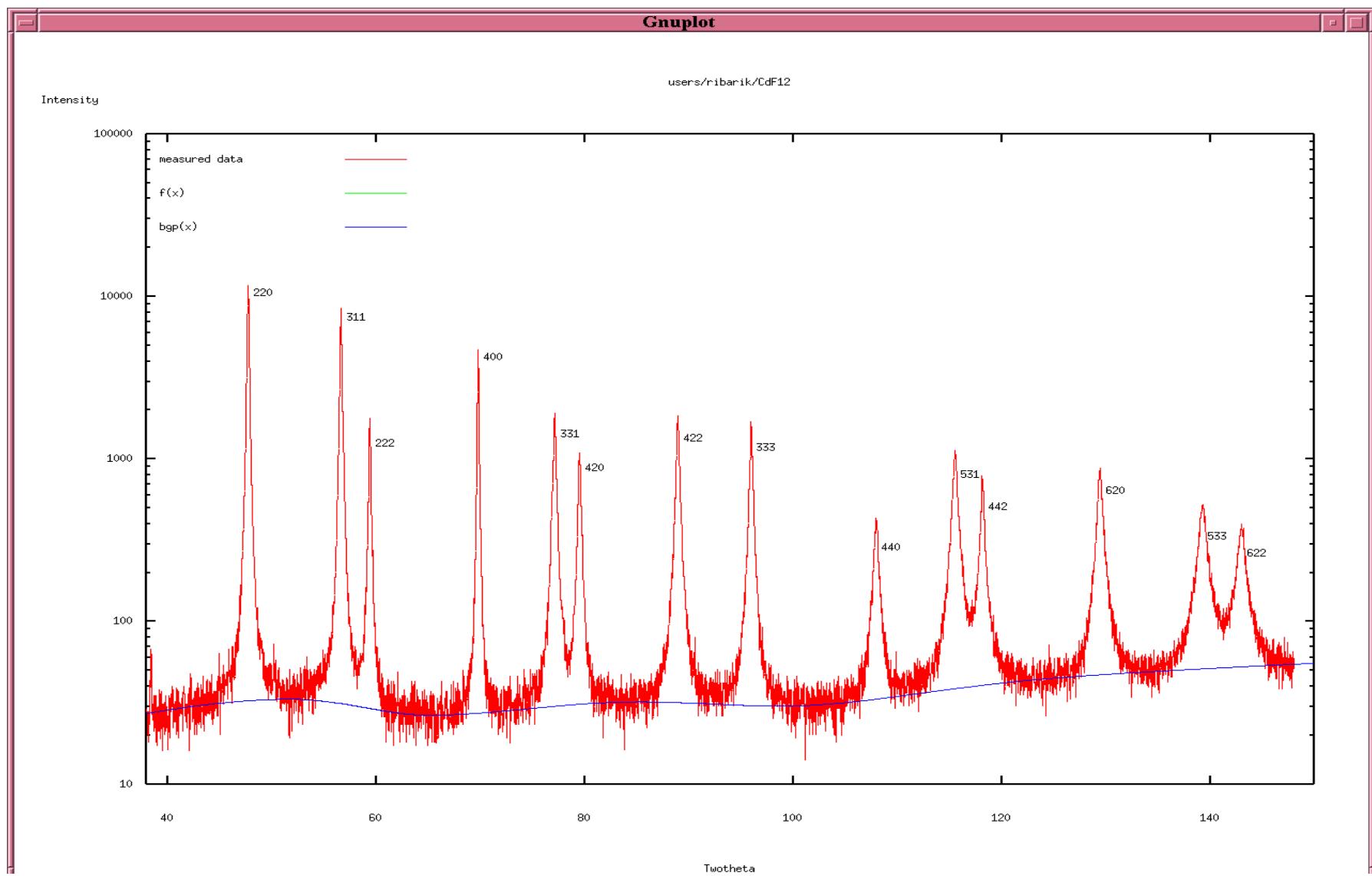


(c)



CdF₂ ball milled for 12 min

G. Ribárik, N. Audebrand, H. Palancher, T. Ungár and D. Louër, *J. Appl. Cryst.* (2005). 38, 912–926



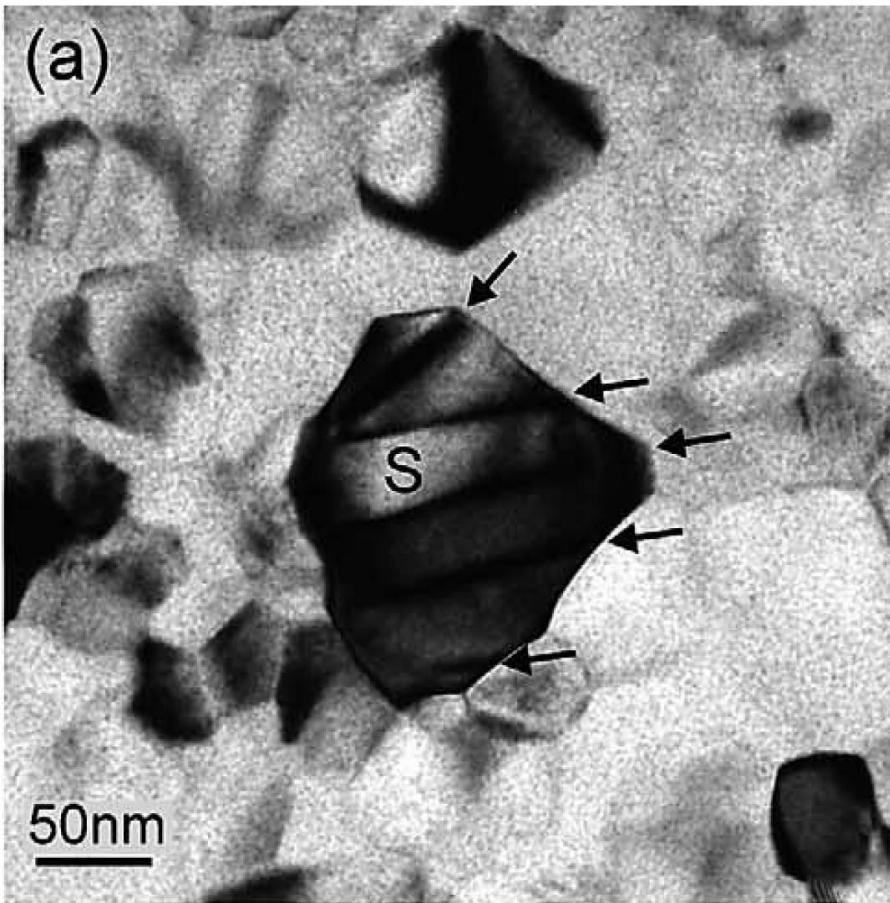
T. Waitz, V. Kazykhanov, H.P. Karnthaler:

Martensitic phase transformations in nanocrystalline NiTi studied by TEM

Acta Materialia 52 (2004) 137–147

Perfect matching along the twin plane

Bright field image of a **twinned grain**.



HRTEM image of a grain containing two **twin related variants**: RT1 & RT2

